**22P203** (Pages: 2) Name......................

Reg.No...................

**SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2023**

(CUCSS - PG)

(Regular/Supplementary/Improvement)

**CC19 MTH2 C08 - TOPOLOGY**

(Mathematics)

(2019 Admission onwards)

**Time: 3 Hours** **Maximum: 30** Weightage

## Part A

**Answer *all* questions. Each carries 1 weightage.**

1. Prove that a subset $A$ of a topological space $X$ is open if and only if it is a nieghbourhood of each of its points.

2. Define projection functions. Prove that projection functions are not closed.

3. Let $(X, \tau)$ be a topological space abd $\mathcal{B}$ is a subfamily of $\tau$. Then prove that $\mathcal{B}$ is a base for $\tau$ if and only if for any $x \in X$ and an open set $G$ containing $x$, there exists $B \in \mathcal{B}$ such that $x \in B$ and $B \subset G$.

4. Prove that every co-finite space is compact.

5. Prove that a space $X$ is locally connected at $x \in X$, if and only if for every nieghbourhood $N$ of $x$ the component of $N$ containing $x$ is a nieghbourhood of $x$.

6. Prove that every closed surjective map is a quotient map.

7. Suppose $y$ is an accumulation point of a subset $A$ of a $T_1$ space X. Then prove that every nieghbourhood of $y$ contains infinitely many points of $A$.

8. Define $T_3$ space. Prove that every $T_3$ space is $T_2$

**($8 \times 1 = 8$ Weightage)**

## Part B

**Answer any *two* questions from each unit. Each carries 2 weightage.**

### Unit 1

9. Let $(X, \tau)$ be a topological space and $S$ is a family of subsets of $X$. Prove that $S$ is a subbase for $\tau$ if and only if $S$ generates $\tau$.

10. Prove that a subset $A$ of a space X is dense in X if and only if, for every non-empty open subset $B$ of X, $A \cap B \neq \phi$. Justify $\mathbb{Q}$ is dense in $\mathbb{R}$.

11. For a subset A of a space $X$, Prove that $\overline{A} = A \cup A'$.

## Unit 2

12. Let $X$ and $Y$ be topological spaces $x \in X$ and $f: X \to Y$ a function. Then prove that f is continuous at $x$ if and only if for every sequence $\{x_n\}$ which converges to $x$ in $X$ in the sequence $\{f(x_n)\}$ converges to $f(x)$ in $Y$.

13. Prove that every quotient space of locally connected space is locally connected.

14. Prove that subset of $\mathbb{R}$ is connected if and only if it is an interval.

## Unit 3

15. Prove that all metric spaces are $T_4$.

16. Prove that every regular Lideloff space is normal.

17. Define heriditary property and prove that regularity is a heriditary property.

**($6 \times 2 = 12$ Weightage)**

## Part C

**Answer any *two* questions. Each carries 5 weightage.**

18. Prove that the product topology on $\mathbb{R}^n$ coincides with the usual topology on it.

19. (a) Let $\mathfrak{C}$ be the collection of connected subsets of a space $X$, such that no two members of $\mathfrak{C}$ are nutually seperated. Prove that $\cup_{c \in \mathfrak{C}} c$ is connected.

    (b) Prove that product of two connected topological space is connected.

20. (a) Prove that every Tyconoff space is $T_3$.

    (b) Let $Y$ be a Hausdroff space. Prove that for any space $X$ and two continuous maps $f$ and $g$ from X to Y, the set $\{x \in X : f(x) = g(x)\}$ is closed in $X$.

21. State and prove Tietze extension theorem.

**($2 \times 5 = 10$ Weightage)**

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