22P204 (Pages: 2) Name: ....

Reg.No: ....

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2023

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MTH2 C09 - ODE AND CALCULUS OF VARIATIONS

(Mathematics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Find a power series solution of the differential equation $y' = y$.
2. Determine the nature of the point $x = 0$ for the differential equation $y'' + \frac{1}{x^2}y' - \frac{1}{x^3}y = 0$.
3. Verify that $e^x = \lim_{b\to\infty} F(a, b, a, \frac{x}{b})$.
4. Describe the phase portrait of the system $\dfrac{dx}{dt} = 0, \ \dfrac{dy}{dt} = 0.$
5. Determine the nature and stability properties of the critical point $(0, 0)$ of the linear autonomous system $\dfrac{dx}{dt} = -3x + 4y, \ \dfrac{dy}{dt} = -2x + 3y.$
6. Determine whether the function $f(x, y) = -x^2 - 4xy - 5y^2$ is positive definite, negative definite or neither.
7. State Sturm comparison theorem.
8. Using Picard's method of successive approximation, solve the initial value problem $y' = y, \ y(0) = 1$ (start with $y_0(x) = 1$).

(8 × 1 = 8 Weightage)

Part B

Answer any ***two*** questions each unit. Each question carries 2 weightage.

UNIT - I

9. Find the general solution of $y'' + xy' + y = 0$ in terms of power series in x .
10. Determine the nature of the point $x = \infty$ for the Euler's equation $x^2y'' + 4xy' + 2y = 0$
11. For the Legendre polynomial $P_n(x)$, prove that $P_n(1) = 1$ and $P_n(-1) = (-1)^n$.

UNIT - II

12. Obtain the two independent solutions of the homogeneous system $\begin{cases} \dfrac{dx}{dt} = x + 2y \\ \dfrac{dy}{dt} = 3x + 2y \end{cases}$ and hence write the general solution of this system. Also show that $x = 3t - 2, y = -2t + 3$ is a particular solution of the nonhomogeneous system $\begin{cases} \dfrac{dx}{dt} = x + 2y + t - 1 \\ \dfrac{dy}{dt} = 3x + 2y - 5t - 2 \end{cases}$ and then write the general solution of this system.

13. For the linear system $\dfrac{dx}{dt} = -x, \quad \dfrac{dy}{dt} = -2y$, (i) find the critical points (ii) find the general solution (iii) find the differential equation of paths and solve it (iv) discuss the stability of the critical point.

14. Verify that (0, 0) is a simple critical point for the system $\dfrac{dx}{dt} = x + y - 2xy, \quad \dfrac{dy}{dt} = -2x + y + 3y^2$ and determine its nature.

UNIT - III

15. State and prove Sturm separation theorem.

16. Show that $f(x, y) = xy^2$ satisfies a Lipschitz condition on any rectangle $a \le x \le b$ and $c \le y \le d$, but does not satisfy a Lipschitz condition on any strip $a \le x \le b$ and $-\infty < y < \infty$.

17. Find the plane curve of fixed perimeter and maximum area.

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. (i) State and prove the orthogonality property of Legendre polynomials.

    (ii) Find the first two terms of the Legender series of $f(x) = \begin{cases} 0 & \text{if } -1 \le x < 0, \\ x & \text{if } 0 \le x \le 1 \end{cases}$.

19. State and prove the orthogonality property of Bessel functions.

20. Find the general solution of $\dfrac{dx}{dt} = 3x - 4y, \quad \dfrac{dy}{dt} = x - y$.

21. Derive Euler's equation for an extremal and find the curve joining the points (x_1, y_1) and (x_2, y_2) that yields a surface of revolution of minimum area when revolved about the x- axis.

(2 × 5 = 10 Weightage)

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