

22U301

(Pages: 2)

Name:

Reg.No:

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2023

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC19U MTS3 B03 / CC20U MTS3 B03 - CALCULUS OF SINGLE VARIABLE - II

(Mathematics - Core Course)

(2019 Admission onwards)

Time : 2.5 Hours

Maximum : 80 Marks

Credit : 4

Part A (Short answer questions)

Answer *all* questions. Each question carries 2 marks.

1. Expand the expression $\ln \left(\frac{x+1}{x-1} \right)^{1/4}$.
2. What is a one-to-one function? Give an example.
3. Define the logarithmic function $f(x) = \log_a(x)$, where $a > 0$ and $a \neq 1$. What are its domain and range?
4. Evaluate $\int_2^4 \frac{1}{\sqrt{4-x}} dx$ and interpret your result geometrically.
5. Show that $\sum_{n=1}^{\infty} (-1)^{n-1}$ is divergent.
6. Use integral test to determine whether $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converges or diverges.
7. State comparison test.
8. State alternating series test.
9. Describe the curves represented by the parametric equations $x = a \cos \theta$ and $y = a \sin \theta$, $a > 0$ with parameter interval $[0, \pi]$.
10. Find the points on the curve $x = t^2 - 4$, $y = t^3 - 3t$ at which the tangent line is either horizontal or vertical.
11. Sketch the region comprising points whose polar coordinates satisfy the condition $1 \leq r < 2$.
12. Sketch the cylinder $y^2 + z^2 = 9$.
13. The point $(\sqrt{3}, -1, 4)$ is expressed in rectangular coordinates. Find its cylindrical coordinates.
14. Find $\bar{\gamma}'(t)$ and $\bar{\gamma}''(t)$ of $\bar{\gamma} = t\bar{i} + t^2\bar{j} + t^3\bar{k}$
15. Define smooth curve.

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer **all** questions. Each question carries 5 marks.

16. Use logarithmic differentiation, find the derivative of the function $y = \sin x^{\tan x}$.
17. Show that $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$.
18. Evaluate $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$
19. Determine whether the sequence $a_n = 3 - \frac{1}{n}$ is monotonic. Is the sequence bounded?
20. Find the radius of convergence and the interval of convergence of $\sum_{n=0}^{\infty} \frac{(-1)^n 2^n x^n}{\sqrt{n+1}}$.
21. Find the area of the region that lies outside the circle $r = 3$ and inside the cardioid $r = 2 + 2\cos\theta$.
22. Find an equation of the plane that passes through the point $(-1, 2, 3)$ and contains the line $x = -1 + 2t$, $y = -2 + 3t$, $z = 3 - t$.
23. Sketch the curve defined by the vector function $\vec{\gamma}(t) = \langle 3 \cos t, -2 \sin t \rangle$, $0 \leq t \leq 2\pi$

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any **two** questions. Each question carries 10 marks.

24. Find the area of the region under the graph of $y = \frac{1}{4+x^2}$ on the interval $[0, 1]$.
25. a) Determine whether the series $\sum_{n=1}^{\infty} \frac{(-2)^{n-1}}{n^2}$ conditionally convergent.
b) Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n}}$ absolutely convergent.
26. Find $\int \cos x^2 dx$.
27. A shell is fired from a gun located on a hill $100m$ above a level terrain. The muzzle speed of the gun is $500m/sec$, and its angle of elevation is 30° .
 - a) Find the range of the shell.
 - b) What is the maximum height attained by shell?
 - c) What is the speed of the shell at impact?

(2 × 10 = 20 Marks)
