21U501 (Pages: 2) Name: ……..………..……..

Reg. No: ………………….

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2023

(CBCSS-UG)

(Regular/Supplementary/Improvement)

CC20U MTS5 B05 – ABSTRACT ALGEBRA

(Mathematics – Core Course)

(2020 Admission onwards)

Time: 2.5 Hours Maximum: 80 Marks

Credit: 4

Section A

Answer ***all*** questions. Each question carries 2 marks.

1. Find the multiplicative inverse of $[7]$ in Z_{15}.
2. Check whether the relation on $\mathbb{R}$ defined by $a \sim b$ if $a \leq b$ is an equivalence relation.
3. Consider the permutation $\sigma = (1,2,3,4)$. Evaluate σ^{100}.
4. Find the number of elements in the set $\{\sigma \in S_4 : \sigma(3) = 3\}$.
5. Find the identity element of the binary operation $\oplus$ on Z, defined by
$$a \oplus b = ab - a - b + 2$$
6. Give an example of a finite non-cyclic group.
7. Show that if every element of a group G is its own inverse, then G is abelian.
8. In $GL_2(\mathbb{R})$, find the order of the element $\begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix}$.
9. Prove that every cyclic group is abelian.
10. Find the number of generators of the cyclic group Z_{25}
11. Show that the function $\varphi: \mathbb{R}^{\times} \to \mathbb{R}^{+}$ defined by $\varphi(x) = |x|$ is a group homomorphism.
12. Find all cosets of the subgroup $4Z$ of $2Z$.
13. Let $\mu = (1,2,4)$ in S_5. Find the index of $\langle \mu \rangle$ in S_5
14. Give an example of a noncommutative ring.
15. Give an example of an integral domain which is not a field

(Ceiling: 25 Marks)

Section B

Answer ***all*** questions. Each question carries 5 marks.

16. Show that the set of all positive rational numbers forms an abelian group under the operation defined by $a * b = \frac{ab}{4}$

17. Let G be a group and H be a subset of G. Prove that H is a subgroup of G if and only if H is non-empty and $ab^{-1} \in H$ for all $a, b \in H$.
18. Let a be an element of a group G. If a has finite order and $k \in Z$, then prove that $a^k = e$, the identity element of G, if and only if $o(a)$ divides k.
19. Prove that the inverse of a group isomorphism is a group isomorphism.
20. If G is an infinite cyclic group then prove that $G \cong Z$.
21. Discuss S_3 and draw its subgroup diagram.
22. Prove that the cancellation laws for multiplication hold in a commutative ring R if and only if R has no divisors of zero.
23. Prove that every field is an integral domain.

(Ceiling: 35 Marks)

Section C (Essay Type)

Answer any ***two*** questions. Each question carries 10 marks.

24. If $n \geq 2$, then prove that the collection of all even permutations of $\{1,2, \dots, n\}$ forms a subgroup of order $\frac{n!}{2}$ of the symmetric group S_n.
25. Prove that any permutation can be written as a product of transpositions.
26. State and prove Cayley's theorem.
27. Define new operations on Z by letting $a \oplus b = a + b - 1$ ans $a \odot b = a + b - ab$ for all $a, b \in Z$. Show that Z is a commutative ring under these operations.

(2 × 10 = 20 Marks)

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