21U502 (Pages: 2) Name: ……..…………..……..

Reg. No: ………………….

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2023

(CBCSS-UG)

(Regular/Supplementary/Improvement)

CC20U MTS5 B06 – BASIC ANALYSIS

(Mathematics – Core Course)

(2020 Admission onwards)

Time: 2.5 Hours Maximum: 80 Marks

Credit: 4

Section A

Answer ***all*** questions. Each question carries 2 marks.

1. Define denumerable sets. Give an example.
2. If z and a are elements in $\mathbb{R}$ with $z + a = a$ then prove that $z = 0$.
3. State the Trichotomy property of set of positive real numbers.
4. If $a \in \mathbb{R}$ is such that $0 \le a < \varepsilon$ for every $\varepsilon > 0$, then prove that $a = 0$.
5. Define supremum of a subset S of R. Give an example of a set which have neither a supremum nor an infimum.
6. Show that $lim(1/n) = 0$.
7. Show that $lim(\frac{2n}{n+1}) = 2$.
8. Show that the sequence $(\sqrt{}\, n)$ is properly divergent.
9. Define a contractive sequence. Give an example.
10. Define a closed subset of the real line. Give an example.
11. Prove that the set $(0,1]$ is not open.
12. Express $-\sqrt{3} - i$ in polar form.
13. Find the reciprocal of $2 - 3i$.
14. What do you mean by the deleted neighbourhood of a point in the complex plane? Give an example.
15. Find the real and imaginary parts of the function $f(z) = z^2 - (2 + i)z$.

(Ceiling: 25 Marks)

Section B

Answer ***all*** questions. Each question carries 5 marks.

16. State and prove Cantor's theorem.
17. State and prove Bernoulli's in equality.
18. Find all $x \in \mathbb{R}$ that satisfy the equation $|x + 1| + |x - 2| = 7$.

19. State and prove Monotone subsequence theorem.
20. Prove that every contractive sequence is a Cauchy sequence.
21. Prove that a subset of $\mathbb{R}$ is closed if and only if it contains all its cluster points.
22. Find the fourth roots of $z = 1 + i$.
23. Find the image of the rectangle with vertices $-1 + i, 1 + i, 1 + 2i$ and $-1 + 2i$.

(Ceiling: 35 Marks)

Section C

Answer any ***two*** questions. Each question carries 10 marks.

24. State and prove nested intervals property.
25. Prove that there exists $x \in \mathbb{R}$ such that $x^2 = 2$.
26. Prove that a subset of $\mathbb{R}$ is open if and only if it is the union of countably many disjoint open intervals in $\mathbb{R}$.
27. Find the image of the triangle with vertices $0, 1 + i$ and $1 - i$ under the mapping $w = z^2$.

(2 × 10 = 20 Marks)

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