21U503 (Pages: 2) Name: ....

Reg.No: ....

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2023

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC20U MTS5 B07 - NUMERICAL ANALYSIS

(Mathematics - Core Course)

(2020 Admission onwards)

Time : 2.00 Hours Maximum : 60 Marks

Credit : 3

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Use algebraic manipulations to show that the function $g(x) = \left(\dfrac{3+x-x^4}{2}\right)^{1/2}$ has a fixed point at p precisely when $f(p) = 0$ where $f(x) = x^4 + 2x^2 - x - 3$
2. Let $f(x) = x^2 - 6$ and $p_0 = 3$. Use Newton's method to find p_3
3. Discuss the disadvantages of Secant method comparing with Newton-Raphson method.
4. Suppose that $x_0 = 1, x_1 = 2, x_2 = 3, x_3 = 4, x_4 = 6$ and $f(x) = e^x$. Determine the interpolating polynomial $P_{1,3,4}(x)$.
5. Using the forward-difference formula approximate the derivative of $f(x) = \sin x$ at $x_0 = 0.5$ by considering $h = 0.1$. Compute the actual error occurred in the approximation.
6. Write down the three-point endpoint formula and three-point midpoint formula to find the approximate derivative of a function $f(x)$ at a given pointx_0.
7. Let $f(x) = \cos \pi x$. Use the midpoint formula and the values of $f(x)$ at $x = 0.25,\ 0.5$ and 0.75 to approximate $f''(0.5)$.
8. Determine the values of h that will ensure an approximation error of less than *0.00002* when approximating $\int_0^\pi \sin x \, dx$ using composite trapezoidal rule.
9. State the fundamental existence and uniqueness theorem for first order ordinary differential equations.
10. Show that the initial value problem $y' = -\frac{2}{t}y + t^2 e^t,\ 1 \le t \le 2,\ y(1) = \sqrt{2}e$ has a unique solution.
11. Use midpoint method to approximate $y(1.2)$ given $y' = (y/t) - (y/t)^2,\ y(1) = 1$.
12. Write the difference equation of the Adams-Bashforth two-step explicit method.

(Ceiling: 20 Marks)

Part B (Short essay questions - Paragraph)

Answer ***all*** questions. Each question carries 5 marks.

13. Use method of false position to find solution of $\sin x - e^{-x} = 0$ for $0 \leq x \leq 1$ accurate to within 10^{-3}.

14. Using Newton's divided difference formula construct an interpolating polynomials of degree three for the data given in the table,

x	-0.1	0.0	0.2	0.3
$f(x)$	5.30	2.00	3.19	1.00

Add $f(0.35) = 0.97$ to the table and construct the interpolating polynomial of degree four.

15. Using Newton's forward-divided-difference formula evaluate $P_4(1.1)$ corresponding to the data given in the table,

x	1	1.3	1.6	1.9	2.2
$f(x)$	0.7652	0.6201	0.4554	0.2818	0.1104

16. Approximate $\int_1^{1.5} x^2 \ln x \, dx$ using Trapezoidal rule. Find a bound for the error using error formula and compare this to the actual error.

17. Compare the Trapezoidal rule and Simpson's rule approximations to $\int_0^2 (1+x)^{-1} \, dx$. Determine the actual error of approximation.

18. Use Euler's method to approximate the solution of the initial value problem $y' = y - t^2 + 1,\ 0 \leq t \leq 2,\ y(0) = 0.5$ with $h = 0.4$. The actual solution is given by $y(t) = (t+1)^2 - 0.5e^t$. Compare the actual error at each step to the error bound.

19. Use Taylor's method of orders two and four to approximate the solution of the initial value problem $y' = \dfrac{y^2 + y}{t},\ 1 \leq t \leq 3,\ y(1) = -2$, with $h = 1$.

(Ceiling: 30 Marks)

Part C (Essay questions)

Answer any ***one*** question. The question carries 10 marks.

20. Use the Bisection method to find solutions accurate to within 10^{-4} for $f(x) = x^3 - 7x^2 + 14x - 6 = 0$ on the interval $[3.2, 4]$.

21. Use Runge-Kutta method of order four to approximate the solution of the initial value problem $y' = -(y+1)(y+3),\ 0 \leq t \leq 2,\ y(0) = -2$, with $h = 0.5$. Compare the results to the actual values.

(1 × 10 = 10 Marks)

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