23P154 (Pages: 2) Name: ....

Reg.No: ....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2023

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC22P MST1 C01 - ANALYTICAL TOOLS FOR STATISTICS – I

(Statistics)

(2022 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part-A

Answer any ***four*** questions. Each question carries 2 weightage.

1. Show that the function $f(x,y,z) = (y+z)^2 + (z+x)^2 + xyz$ has no maximum or minimum value.
2. What is directional derivaties and total derivaties of a function? Explain.
3. Find the minimum value of $f(x,y) = x^2 + 5y^2 - 6x + 10y + 6$.
4. Prove that, if a function $f(z)$ is analytic within and on a closed contour c and a is any point lying in it, then $f^{|}(a) = \frac{1}{2\pi i}\int \frac{f(z)}{(z-a)^2} dz$.
5. Establish Lioville's theorem.
6. Prove that the function $e^{1/z}$ has an isolated singularity at $z = 0$.
7. Find the residue of $\frac{1}{(z^2+1)^3}$ at $z = i$.

(4 × 2 = 8 Weightage)

Part-B

Answer any ***four*** questions. Each question carries 3 weightage.

8. Prove that the function $u(x,y) = e^{-x} siny$ is harmonic and find the corresponding analytic function.
9. State and prove Cauchy-Reimann condition for an analytic function in polar form.
10. Show that $\int_0^\infty \frac{dx}{x^2+1} = \frac{\pi}{2}$.
11. Evaluate

    a) $\int_0^\infty t^3 e^{-t} sint\ dt$ b) $\int_0^\infty \frac{e^{-t} - e^{-3t}}{t}\ dt$
12. Find the inverse Laplace transform of $\frac{1}{s^2(s+1)^2}$ and $\frac{1}{(s^2+1)^2}$.
13. Find the Fourier series expansion of $f(x) = x\ sin\ x\ ; -\pi < x < \pi$.
14. Find the finite $cosine$ transform of $f(x) = (1 - \frac{x}{\pi})^2; 0 < x < \pi$.

(4 × 3 = 12 Weightage)

Part-C

Answer any ***two*** questions. Each question carries 5 weightage.

15. What is Poisson's integral formula? Prove Poisson integral formula.

16. (a) State and Prove Laurent's theorem.

    (b) If $0 < |z-1| < 2$, then express $f(z) = \frac{z}{(z-1)(z-3)}$ in a series of positive and negative powers of $(z-1)$.

17. (a) State and prove Jordan's lemma.

    (b) State and prove the Cauchy Residue theorem.

18. (a) Solve the initial value problem, $y'' + 4y' + 3y = 0, y(0) = 3, y'(0) = 1.$

    (b) Solve $y'' + 2y' + 5y = 0, y(0) = 2, y'(0) = -4.$

(2 × 5 = 10 Weightage)

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