22P362 (Pages: 2) Name: ………………………

Reg. No: ....………………....

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2023

(CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19P ST3 C12 / CC22P MST3 C12 – STOCHASTIC PROCESSES

(Statistics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

PART A

Answer any ***four*** questions. Each question carries 2 weightage.

1. Define Stochastic process. Explain the classification with the help of examples.
2. Explain one dimensional Random Walk.
3. Discuss Birth and Death process.
4. What are the properties of a Poisson process?
5. Define renewal equation. Show that a renewal equation is satisfied by a renewal equation.
6. Establish a necessary and sufficient condition satisfied by the recurrence state of a Markov Chain.
7. Define Brownian motion process.

(4 × 2 = 8 Weightage)

PART B

Answer any ***four*** questions. Each question carries 3 weightage.

8. Derive the Chapman-Kolmogorov equation for continuous time Markov Chain.
9. For a Poisson process with rate λ, show that interarrival time between two successive events is Exponentially distributed with mean $1/\lambda$
10. Describe renewal reward process.
11. Explain:
    (a) Semi-Markov process
    (b) Non homogeneous Poisson process
    (c) One dimensional Random Walk
12. Define branching process. Find the mean and variance of the G.W branching process.
13. Prove that the interval between two successive occurrences of a Poisson process follow exponential distribution.
14. State and prove elementary renewal theorem.

15. If $\{(t)\}$ is a Poisson process derive auto-correlation between (t) and $N(t+s), t, s > 0$.

(4 × 3 = 12 Weightage)

PART C

Answer any ***two*** questions. Each question carries 5 weightage.

16. Explain the basic characteristics of queues. Obtain the steady state probability distribution of M/M/1 model.
17. Derive Pollock-Kinchins formulae. Explain the hitting time of a Brownian motion process.
18. State and prove central limit theorem on renewal process. Stochastic process having independent increment is a Markov process. Is the converse true?
19. (a) State and prove Ergodic theorem of Markov chain.

    (b) Determine the nature of the states of the Markov Chain

    $$\begin{bmatrix} 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0 \\ 0.5 & 0 & 0.5 \end{bmatrix}$$

    (c) Describe Gambler's ruin problem.

(2 × 5 = 10 Weightage)

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