22P301 (Pages: 2) Name: ..……………………..

Reg. No: ..…………………..

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2023

(CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19P MTH3 C11 – MULTIVARIABLE CALCULUS AND GEOMETRY

(Mathematics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Prove that every span is a vector space.
2. Define an independent set and prove that no independent set contains the null vector.
3. State true or false and justify your statement.

   A vector space can have more than one basis with different dimensions.
4. Define parametrization of a parametrized curve. Give an example.
5. Prove that the total signed curvature of a closed curve is an integer multiple of 2π.
6. Show that any open disc in the $xy-$ plane is a surface.
7. Define surface patch.
8. Show that the second fundamental form of a plane is zero.

(8 × 1 = 8 Weightage)

PART B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT I

9. Prove that BA is linear if A and B are linear transformations. Also prove that A^{-1} is linear and invertible.
10. Let Ω be the set of all invertible linear operators on R^n. Prove that Ω is an open subset of $L(R^n)$.
11. Define contraction. Prove that if X is a complete metric space, and if Φ is a contraction of X into X, then there exists one and only one $x\epsilon\ X$ such that $\Phi(x) = x$.

UNIT II

12. Show that a parametrized curve has a unit – speed reparametrization if and only if it is regular.
13. Prove that the curvature of a circular helix is a constant.

14. Show that every plane in R^3 is a surface with an atlas consisting of a single surface patch.

UNIT III

15. Calculate the first fundamental forms of the surface

    $\sigma(u,v) = (\sin u \ \sinh v, \sinh u \ \cosh v, \sinh u)$

16. Define curvature, normal curvature and geodesic curvature. Explain in brief the relationship between them.

17. Differentiate between Gauss and Weingarten maps.

(6 × 2 = 12 Weightage)

PART C

Answer any ***two*** questions. Each question carries 5 weightage.

18. Establish the relationship between the total derivative and partial derivatives of a map $\boldsymbol{f}$ from an open set $\boldsymbol{E} \subset \boldsymbol{R^n}$ into $\boldsymbol{R^m}$ at a point $\boldsymbol{x} \in \boldsymbol{E}$.

19. State and prove Inverse function theorem.

20. a) Give an example to show that the reparametrization of a closed curve need not be closed.

    b) Show that if a curve $\boldsymbol{\gamma}$ is T_1 – periodic and T_2 – periodic, then it is $(k_1T_1 + k_2T_2)$ – periodic for any integers k_1 and k_2.

21. a) Prove that two curves which touch each other at a point $\boldsymbol{P}$ of a surface (i.e which intersect at $\boldsymbol{P}$ and have parallel tangent vectors at $\boldsymbol{P}$) have the same normal curvature at $\boldsymbol{P}$.

    b) State and prove Meusnier's Theorem.

(2 × 5 = 10 Weightage)

****