22P303 (Pages: 2) Name: ....

Reg.No: ....

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2023

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MTH3 C13 - FUNCTIONAL ANALYSIS

(Mathematics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part A

Answer any ***all*** questions. Each question carries 1 weightage.

1. Show that unit ball of a linear space E is a convex set.
2. Define a complete system with example.
3. Define projection of x in H onto L, where L is a closed subspace of H.
4. If E is a closed subspace of a Hilbert space M and let codim$E = 1$, then show that dim $E^{\perp} = 1$.
5. State Hahn-Banach Theorem. Show that for all $x_0 \in X$ there exists $f_0 \in X^* - \{0\}$ such that $f_0(x_0) = \|f_0\| \cdot \|x_0\|$
6. State Arzela theorem.
7. Define norm convergence, strong convergence and weak convergence.
8. If A and B are invertible opertaors then show that AB is ivertible and $(AB)^{-1} = B^{-1}A^{-1}$

(8 × 1 = 8 Weightage)

Part B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT - I

9. Prove that the dimension of E/E_1 is n if and only if there exist $x_1, x_2, \ldots, x_n$ linearly independent vectors relative to E_1 such that for every $x \in E$ there exist unique set of numbers $a_1, a_2, \ldots a_n$ and a unique vector $y \in E_1$ such that $x = \sum_{i=1}^{n} a_i x_i + y$
10. Let X_0 be a closed subspace of X.. Verify X/X_0 is a normed space together with the norm defined by $||[x]|| = \inf_{y \in X_0} ||x - y||$
11. Let E be a normed space. Prove that there exists a complete normed space E^1 and a linear operator $T : E \to E^1$ such that $||Tx|| = ||x||$ for all $x \in E$. Also prove image T is dense in E^1.

UNIT - II

12. State and prove Cauchy Schwartz inequality.

13. Prove that for any $x \in H$ and any orthonormal system $\{e_i\}_1^{\infty}$, there exists a $y \in H$ such that $y = \sum_{i=1}^{\infty} < x, e_i > e_i$

14. If E is a closed subspace of $H$and codim $E = 1$, then show that the subspace $E^{\perp}$ is 1-dimensional.

UNIT - III

15. Prove that the dual space of c_0 is l_1.

16. Prove that $K(X \to Y)$ is a closed linear subspace of $L(X \to Y)$.

17. If $A : X \to Y$ is compact then prove that $A^* : Y^* \to X^*$ is compact.

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. State and Prove Minkowski's ineqality for sequences

19. Show that the Hilbertspace is seperable if and only if there exist a complete orthonormal system $\{e_i\}_{i\geq 1}$

20. State and prove Riesz representation theorem.

21. Let X be a normed space and let Y be a complete normed space. Then prove that $L(X \to Y)$is a Banach Space.

(2 × 5 = 10 Weightage)

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