22P304 (Pages: 2) Name……………………..

Reg. No…………………..

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2023

(CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19P MTH3 C14 – PDE AND INTEGRAL EQUATIONS

(Mathematics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Solve the equation $-yu_x + xu_y = u$ subject to the initial condition $u(x, 0) = \phi(x)$.
2. Which are the condition for a first order quasilinear partial differential equation to have a unique solution?
3. Derive the general solution of one dimensional heat equation.
4. Define Cauchy problem for quasilinear equations.
5. State the mean value principle.
6. If $y''(x) = F(x)$ and y satisfies the conditions $y(0) = 0$ and $y(1) = 0$, show that $y(x) = \int_0^1 K(x,\xi)F(\xi)d\xi$, where $K(x, \xi) = \begin{cases} \xi(x-1), x > \xi \\ x(\xi-1), x < \xi \end{cases}$.
7. Define separable kernel and give an example of it.
8. Determine the resolvent kernel associated with $K(x, \xi) = x + \xi$ in (0, 1) in the form of power series in λ obtaining the first three terms.

(8 × 1 = 8 Weightage)

PART B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT-1

9. Show that the Cauchy problem $u_x + u_y = 1$, $u(x, x) = x$ has infinitely many solutions.
10. Find a coordinate system $s = s(x, y)$, $t = t(x,y)$ that transforms the equation $u_{xx} + 2u_{xy} + 15u_{yy} = 0$ into canonical form.
11. Consider the problem

    $u_{tt} - c^2 u_{xx} = 0, \quad -\infty < x < \infty$ ------ (1)

    $u(x, 0) = f(x), \quad u_t(x, 0) = g(x), \quad -\infty < x < \infty$ ------ (2)

    Fix $T > 0$. Prove that the Cauchy problem (1), (2) in the domain $-\infty < x < \infty$, $o \le t \le T$ is well- posed for $f \in C^2(\Re)$, $g \in C'(\Re)$.

UNIT-2

12. Solve the problem: $u_t - u_{xx} = 0,\ 0 < x < \pi,\ t > 0$

$u(0, t) = u(\pi, t) = 0,\ t \geq 0$

$$u(x, 0) = f(x) = \begin{cases} x, & 0 \leq x \leq \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} \leq x \leq \pi \end{cases}.$$

13. Prove that the Neumann problem has a unique solution.

$u_{tt} - c^2 u_{xx} = F(x, t),\ 0 < x < L,\ t > 0$

$u_x(0, t) = a(t),\ u_x(L, t) = b(t),\ t \geq 0$

$u(x, 0) = f(x),\ 0 \leq x \leq L$

$u_t(x, 0) = g(x),\ 0 \leq x \leq L,$

14. Derive the equations for Poisson's kernel.

UNIT-3

15. Transform the problem $y'' + y = x;\ y(0) = 0,\ y'(1) = 0$ to a Fredholm integral equation Using Green's function.
16. Solve the Fredholm integral equation by iterative method: $y(x) = 1 + \lambda \int_0^1 (1 - 3x\,\xi)\, y(\xi)\, d\xi$.
17. Formulate the integral equation corresponding to the differential equation $x^2y'' + xy' + (\lambda x + 3) = 0$.

(6 × 2 = 12 Weightage)

PART C

Answer any ***two*** questions. Each question carries 5 weightage.

18. a. Use the method of characteristics strips solve the equation $p^2 + q^2 = 2A$ subject to the condition $u(x, 2x) = 1$.

    b. Write the canonical form of the equation $u_{xx} + 4u_{xy} + u_x = 0$.
19. Solve the problem $u_{tt} - u_{xx} = t^3,\ -\infty < x < \infty,\ t > 0$ subject to the conditions.

$u(x, 0) = x + \sin x,\ -\infty < x < \infty,$

$u_t(x, 0) = 0,\ -\infty < x < \infty,$

20. Solve using separation of variables:

$u_{tt} - 9u_{xx} = 0,\ 0 < x < 2\ \ t > 0$

$u_x(0, t) = u_x(2, t) = 0,\ t \geq 0$

$u(x, 0) = \cos^2 \pi x,\ 0 \leq x \leq 2$

$u_t(x, 0) = \sin^2 \pi x,\ 0 \leq x \leq 2.$

21. Determine the characteristic values of λ and the corresponding characteristic functions of the equation $y(x) = F(x) + \lambda \int_0^{2\pi} \sin(x - \xi)\, y(\xi)\, d\xi$ considering all possible cases.

(2 × 5 = 10 Weightage)

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