22U364S (Pages: 2) Name: …………………

Reg: No: ………………

THIRD SEMESTER B.Voc. DEGREE EXAMINATION, NOVEMBER 2023

(Information Technology)

CC18U GEC3 ST08 – PROBABILITY DISTRIBUTIONS

(2018 to 2020 Admissions – Supplementary)

Time: Three Hours Maximum: 80 Marks

PART A

Answer ***all*** questions. Each question carries 1 mark.

Fill in the blanks:

1. X is a Poisson random variable with mean 5. Then V(X) = ………...
2. Binomial distribution B(n,p) is symmetric when p= ………...
3. In the expansion of $M_x(t)$ the coefficient of $\frac{t^r}{r!}$ is ………...
4. For a normal distribution, $\beta_1 = \cdots$
5. The propounder of CLT for i.i.d random variables is ………...

Write true or false:

6. Moment generating function exists for all distributions.
7. Cov(X, Y) = 0 implies that X and Y are independent.
8. Normal curve is assymetric.
9. Gamma distribution possesses lack of memory property.
10. If $X \sim N(\mu, \sigma^2)$, then $\mu_4 = 3\sigma^2$.

(10 × 1 = 10 Marks)

PART B

Answer any ***eight*** questions. Each question carries 2 marks.

11. Define characteristic function of a random variable.
12. List the properties of mathematical expectation.
13. Define marginal density.
14. In a binomial distribution consisting of 5 independent trails, probabilities of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the parameter ‘p’ of the distribution.
15. Define joint probability mass function.
16. Define distribution function of bivariate random variable.
17. Obtain the mean of a continuous uniform distribution.
18. Define moment measures of skewness.
19. Show that m.g.f of a sum of n independent random variables is equal to the product of their m.g.f’s.
20. Define standard normal distribution.

21. Define beta distribution of first kind.
22. Define convergence in probability.

(8 × 2 = 16 Marks)

PART C

Answer any ***six*** questions. Each question carries 4 marks.

23. Establish the relationship between raw moments and central moments.
24. A coin is tossed until a head appears. What is the expectation of the number of tosses required?
25. State and prove Cauchy-Schwartz inequality.
26. State and prove multiplication theorem on expectation.
27. State and prove lack of memory property.
28. Find K so that $f(x) = \begin{cases} k(x^2+y^2), 0 \le x \le 1, 1 \le y \le 4 \\ 0, else\ where \end{cases}$ will be a bivariate probability density function.
29. A random variable X has probability mass function $f(x) = 2^{-x}, x = 1,2,3, \ldots$. Find its mean and variance.
30. State the chief characteristics of normal distribution.
31. Define Gamma distribution. Obtain its moment generating function.

(6 × 4 = 24 Marks)

PART D

Answer any ***two*** questions. Each question carries 15 marks.

32. If μ_r is the r^{th} central moment of the binomial distribution B(n, p) prove that $\mu_{r+1} = pq\left[\frac{d\mu_r}{dp} + nr\mu_{r-1}\right]$. Hence obtain μ_2.
33. (a) Define conditional mean and variance.

    (b) If the joint probability mass function of X and Y is $f(x,y) = \frac{x+3y}{24}, x = 1,2\ \ y = 1,2$.
    Find E{X/Y=2} and Var{X/Y=2}.
34. (a) State and Prove Bernoulli's weak law of large numbers.

    (b) State Lideberg – Levy CLT.
35. (a) State and prove Chebychev's inequality.

    (b) If E(X) = 3, E(X^2) = 13, use chebychev's inequality to find a lower bound for
    P(-2 < X <8).

(2 × 15 = 30 Marks)

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