

**FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2024**

(CBCSS - UG)

**CC19U MTS1 B01 / CC20U MTS1 B01 - BASIC LOGIC AND NUMBER THEORY**

(Mathematics - Core Course)

(2019 to 2023 Admissions - Supplementary/Improvement)

Time : 2.5 Hours

Maximum : 80 Marks

Credit : 4

**Part A (Short answer questions)**Answer *all* questions. Each question carries 2 marks.

1. Define conditional statement and give example.
2. Determine whether  $[p \wedge (p \rightarrow q)] \rightarrow q$  is a tautology.
3. Verify  $\sim (\sim p \vee q) \equiv p \wedge \sim q$ .
4. Negate each quantified propositions.
  - a) Every computer is a 16-bit machine.
  - b) Some girls are blondes.
5. Test the validity of the argument
 
$$\begin{array}{l} p \vee q \\ q \vee r \\ \sim r \\ \hline \therefore p \end{array}$$
6. Prove that  $\sqrt{ab} \leq \frac{a+b}{2}$
7. Define recursively the number sequence 0, 3, 9, 21, 45, .....
8. Are the integers 6 and 35 relatively prime?
9. Can every integer greater than or equal to 2 can be decomposed into primes.
10. If  $ac \equiv bc \pmod{m}$  and  $(c, m) = 1$ , then show that  $a \equiv b \pmod{m}$ .
11. Define solution of linear congruence with an example.
12. Using divisibility test determine whether 30587 and 648 are divisible by 8.
13. Without using Wilsons theorem verify that  $(p - 1)! \equiv -1 \pmod{p}$  for  $p = 5$ .
14. State Fermat's Little Theorem.
15. Define Euler's phi function and compute  $\phi(15)$ .

**(Ceiling: 25 Marks)**

**Part B (Paragraph questions)**

Answer *all* questions. Each question carries 5 marks.

16. Explain:
  - a) Vacuous proof.
  - b) Direct proof.
  - c) Proof of contrapositive.
  - d) Proof by contradiction.
  - e) Proof by cases.
17. Prove that there is no polynomial  $f(x)$  with integral coefficients that will produce primes for all integers  $n$ .
18. Using the Euclidean Algorithm, Find the gcd of 1024, 1000.
19. Find the canonical decomposition of 2520.
20. Using recursion, find the lcm of 12, 18, 20, 28?
21. Twenty three weary travelers entered the outskirts of a lush and beautiful forest. They found 63 equal heaps of plantains and seven single fruits and divided them equally. Find the number of fruits in each heap.
22. Show that  $19^{19}$  cannot be expressed as the sum of the cube of an integer and the fourth power of another integer.
23. Solve the linear congruence  $15x \equiv 7 \pmod{13}$ .

**(Ceiling: 35 Marks)**

**Part C (Essay questions)**

Answer any *two* questions. Each question carries 10 marks.

24. State and prove division algorithm.
25. There are infinitely many primes of the form  $4n + 3$ .
26. a) Using inverses, find the incongruent solution of  $48x \equiv 39 \pmod{17}$ .  
b) Using congruences solve  $3x + 4y = 5$ .
27. a) Using Euler's theorem find the remainder when  $25^{2550}$  is divided by 18.  
b) Solve the linear congruence  $7x \equiv 8 \pmod{10}$ .

**(2 × 10 = 20 Marks)**

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