

## SECOND SEMESTER UG DEGREE EXAMINATION, APRIL 2025

(FYUGP)

## CC24UMAT2MN103 - ANALYSIS AND SOME COUNTING PRINCIPLES

(Mathematics - Minor Course)

(2024 Admission - Regular)

Time: 2.0 Hours

Maximum: 70 Marks

Credit: 4

**Part A** (Short answer questions)Answer **all** questions. Each question carries 3 marks.

1. Check if the alternating series  $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2}$  using the alternating series test. [Level:3] [CO2]
2. Write the first five terms of the sequence  $\{a_n\} = \frac{10n}{n+1}$ . [Level:2] [CO1]
3. Describe the Limit Comparison Test. [Level:1] [CO2]
4. Define the following terms: (i) Disk (ii) Neighbourhood [Level:1] [CO3]
5. What is the polar form of a complex number  $z = x + iy$ ? [Level:1] [CO3]
6. Solve the equation  $\omega^2 = i$ . [Level:2] [CO3]
7. Define derivative of complex function at a point  $z_0$ . [Level:1] [CO4]
8. Define entire function and give an example. [Level:1] [CO4]
9. Describe the sample space where a coin is tossed three times and the sequence of heads and tails is recorded. [Level:2] [CO5]
10. A group of 50 students is to be divided into 8 teams. What is the minimum number of students that must be in one team, according to the pigeonhole principle? [Level:3] [CO5]

**(Ceiling: 24 Marks)****Part B** (Paragraph questions/Problem)Answer **all** questions. Each question carries 6 marks.

11. Determine whether the following sequences are monotonic [Level:3] [CO1]
  - (a)  $\{a_n\} = \frac{n^2}{2^n - 1}$  (b)  $\{a_n\} = 5 - \frac{1}{n}$ .
12. Describe a  $p$ -Series and a harmonic series and explain its convergence. [Level:3] [CO2]

13. Describe the set of points  $z$  in the complex plane that satisfy the given equations : [Level:3] [CO3]  
 (a)  $|z - 1| = 1$  (b)  $|z| = \operatorname{Re}(z)$
14. Evaluate the following powers of  $i$ . [Level:2] [CO3]  
 (a)  $i^{11}$  (b)  $i^{208}$  (c)  $i^{119}$  (d)  $i^{100}$
15. Show that the given functions are not analytic at any point: [Level:2] [CO4]  
 (a)  $f(z) = \operatorname{Re}(z)$  (b)  $f(z) = y + ix$
16. Compute  $\lim_{z \rightarrow 1+i} (z^2 + i)$ . [Level:2] [CO4]
17. (a) From a group of 15 people, how many ways can a team of 4 people be selected if at least 2 of them must be women? [Level:3] [CO5]  
 (b) In how many ways can a 6-card hand be dealt from a deck of 52 cards?
18. (a) A password consists of 2 letters followed by 3 digits. The letters can be any of the 26 alphabetic characters, and the digits can be any of the 10 digits (0-9). [Level:3] [CO5]  
 How many different possible passwords can be formed?  
 (b) Identify the number of distinguishable "words" that can be formed from the letters of MISSISSIPPI.

(Ceiling: 36 Marks)

**Part C** (Essay questions)

Answer any *one* question. The question carries 10 marks.

19. (a) Find the sequence of partial sums  $S_1, S_2, S_3, S_4, S_5$  for the series [Level:3] [CO2]  

$$\sum_{i=1}^{\infty} \left( \frac{3}{2^n - 1} \right).$$
  
 (b) Using the  $n$ th-term test, determine whether the series converges or diverges.  
 (i)  $\sum_{n=0}^{\infty} 2^n$   
 (ii)  $\sum_{n=0}^{\infty} \frac{1}{n}$
20. (a) Verify that the function  $u(x, y) = xy + x + 2y$  is harmonic in an appropriate domain  $D$ . [Level:2] [CO4]  
 (b) Find the harmonic conjugate of  $u$ .  
 (c) Find an analytic function  $f(z) = u(x, y) + iv(x, y)$

(1 × 10 = 10 Marks)

\*\*\*\*\*