24U205 (Pages: 2) Name : ....

Reg. No : ....

SECOND SEMESTER UG DEGREE EXAMINATION, APRIL 2025

(FYUGP)

CC24UMAT2MN104 - GRAPH THEORY AND AUTOMATA

(Mathematics - Minor Course)

(2024 Admission - Regular)

Time: 2.0 Hours Maximum: 70 Marks

Credit: 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 3 marks.

1. Given a subgraph $G = (V, E)$, explain what is meant by a subgraph of G. Provide an example using a graph with 4 vertices. [Level:2] [CO1]
2. Define a Graph. Draw a graph with 5 vertices and 7 edges. [Level:1] [CO1]
3. Define a cycle. What is the length of a cycle in a graph? If a cycle has 6 vertices, what is its length? [Level:2] [CO1]
4. State Dirac's theorem. [Level:1] [CO2]
5. Show that there is a path between any two distinct vertices in a connected graph. [Level:1] [CO2]
6. Find the chromatic number of a complete bipartite graph $K_{m,n}$. Justify your answer. [Level:2] [CO4]
7. Verify Euler's formula of planar graphs with an example. [Level:2] [CO4]
8. Consider the grammar $G = (N, T, P, \sigma)$, where $N = \{\sigma\}$, $T = \{a, b\}$, and $P = \{\sigma \to a\sigma b, \sigma \to ab\}$. Determine if the words $abba$ and $abab$ belongs to $L(G)$. [Level:2] [CO5]
9. Define grammar. [Level:1] [CO5]
10. Define equality of two words. [Level:2] [CO5]

(Ceiling: 24 Marks)

Part B (Paragraph questions/Problem)

Answer ***all*** questions. Each question carries 6 marks.

11. (a) Define a bipartite graph. Provide an example of a bipartite graph with 7 vertices. [Level:2] [CO1]
    (b) Find the number of vertices and edges in a complete bipartite graph $K_{m,n}$.
12. (a) Explain isomorphic graphs with an example. [Level:2] [CO1]
    (b) Prove or disprove: "If two graphs have the same number of vertices and edges, they are necessarily isomorphic." Justify your answer with an example.

13. Draw the graph G represented by the given adjacency matrix $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$. [Level:2] [CO1]

14. Write an algorithm for finding an Eulerian circuit in an Eulerian graph. [Level:2] [CO3]

15. Let P_n denote the path $v_0 - v_1 - v_2 - \dots\dots\dots - v_n$ of length n connecting the vertices $v_0, v_1, v_2, \dots\dots, v_n$ in a simple graph, where $n \geq 0$. Find the independent subsets of vertices in the path when $n = 0, 1, 2$. [Level:2] [CO2]

16. Prove that $K_{3,3}$ is non planar. [Level:2] [CO4]

17. (a) Define a tree, a pendant vertex and draw a tree with 8 vertices among which at least 3 are pendant vertices. [Level:2] [CO4]

(b) Determine the number of edges in a a tree with n vertices.

18. (a) Define a language [Level:2] [CO5]

(b) $L = \{x \in \sum^* : x \text{ } begins \text{ } with \text{ } and \text{ } ends \text{ } in \text{ } b\}$ is a language L over $\sum = \{a, b\}$. Find five words in this language.

(Ceiling: 36 Marks)

Part C (Essay questions)

Answer any ***one*** question. The question carries 10 marks.

19. (a) Explain a spanning tree with an example. How many edges does a spanning tree each of a K_n and a $K_{m,n}$ have. [Level:2] [CO4]

(b) Draw a graph with the following vertices and edges. Vertices: A, B, C, D and Edges with weights: $AB - (3), AC - (1), BC - (7), BD - (5), CD - (2)$. Using Kruskal's algorithm, find the MST and its total weight. List the edges.

20. Draw the transition diagram of the FSA $M = (S, A, I, f, s_0)$, where $S = \{s_0, s_1, s_2, s_3, s_4\}$, $A = \{s_2\}, I = \{a, b, c\}$ and f is defined by the following table [Level:2] [CO5]

S, I	a	b	c
s_0	s_1	s_2	s_3
s_1	s_4	s_2	s_3
s_2	s_1	s_4	s_3
s_3	s_1	s_2	s_4
s_4	s_4	s_4	s_4

(1 × 10 = 10 Marks)

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