24U203 (Pages: 2) Name : ....

Reg. No : ....

SECOND SEMESTER UG DEGREE EXAMINATION, APRIL 2025

(FYUGP)

CC24UMAT2MN105 - VECTOR SPACES AND LINEAR TRANSFORMATIONS

(Mathematics - Minor Course)

(2024 Admission - Regular)

Time: 2.0 Hours Maximum: 70 Marks

Credit: 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 3 marks.

1. Define span of a set of vectors. Give a spanning set for R^3. [Level:2] [CO1]

2. Determine whether the statement is true or false. [Level:2] [CO1]

   a) The set of all polynomials of degree n is a subspace of $F(-\infty, \infty)$.

   b) The set of all polynomials of degree $\leq n$ is a subspace of $F(-\infty, \infty)$.

3. Show that the functions $f_1 = 3\sin^2 x$, $f_2 = 2\cos^2 x$ and $f_3 = 6$ form a linearly dependent set in $F(-\infty, \infty)$. [Level:2] [CO2]

4. What do you mean by a finite dimensional and infinite dimensional vector space? Give an example for each. [Level:1] [CO2]

5. Show that the standard unit vectors in R^n are linearly independent. [Level:2] [CO2]

6. Write standard matrices for the projections onto the xy-plane, the yz-plane, and xz-plane in R^3. [Level:1] [CO3]

7. Write standard matrices for the compression and expansion in R^2. [Level:1] [CO3]

8. Find $det(A)$ given that $p(\lambda) = \lambda^2 - 2\lambda - 3$ as its characteristic polynomial. [Level:3] [CO4]

9. Confirm by multiplication that x is an eigenvector of A, and find the corresponding eigenvalue, where [Level:2] [CO4]

   A= $\begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix}$ and x= $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.

10. Define Eigenspace of a matrix A. [Level:1] [CO4]

(Ceiling: 24 Marks)

Part B (Paragraph questions/Problem)

Answer ***all*** questions. Each question carries 6 marks.

11. Show that lines through the origin and planes through the origin are subspaces of R^3. [Level:3] [CO1]

12. Show that R^n is a vector space. [Level:2] [CO1]

13. Explain why the vectors $v_1 = (2,0,-1)$, $v_2 = (4,0,7)$ and $v_3 = (-1,1,4)$ form a basis for R^3. [Level:3] [CO2]

14. The vectors $v_1 = (1,2,1)$, $v_2 = (2,9,0)$ and $v_3 = (3,3,4)$ form a basis for R^3. Find the coordinate vector of $v = (5,-1,9)$ relative to the basis $S = \{v_1, v_2, v_3\}$. [Level:3] [CO2]

15. Let $T_1 : R^2 \to R^2$ be the reflection on the y-axis and $T_2 : R^2 \to R^2$ be the reflection on the x-axis. Determine whether the operators T_1 and T_2 commute. [Level:2] [CO3]

16. Find the standard matrix for the operator on R^2 that first reflects on the line y = x and then shears by a factor 2 in the x direction. Sketch the image of unit square under this operator. [Level:3] [CO3]

17. Find a matrix P that diagonalizes [Level:3] [CO4]

$$A=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

18. Confirm that P diagonalizes A, and then Compute A^{11}, where [Level:3] [CO4]

$$A=\begin{bmatrix} -1 & 7 & -1 \\ 0 & 1 & 0 \\ 0 & 15 & -2 \end{bmatrix} \text{ and } P=\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 5 \end{bmatrix}$$

(Ceiling: 36 Marks)

Part C (Essay questions)

Answer any ***one*** question. The question carries 10 marks.

19. Find a basis for and the dimension of the solution space of the homogeneous system. [Level:3] [CO2]

$$x_1 - 4x_2 + 3x_3 - x_4 = 0$$
$$2x_1 - 8x_2 + 6x_3 - 2x_4 = 0$$

20. Show that the operator T:$R^2 \to R^2$ defined by the equations. [Level:3] [CO3]

$w_1 = 2x_1 + x_2$
$w_2 = 3x_1 + 4x_2$
is one-to-one , and find $T^{-1}(w_1, w_2)$.

(1 × 10 = 10 Marks)

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