24U202

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Name : ....

Reg. No : ....

SECOND SEMESTER UG DEGREE EXAMINATION, APRIL 2025

(FYUGP)

CC24UMAT2MN102 - CALCULUS AND MATRIX ALGEBRA

(Mathematics - Minor Course)

(2024 Admission - Regular)

Time: 2.0 Hours

Maximum: 70 Marks

Credit: 4

Part A (Short answer questions)

Answer *all* questions. Each question carries 3 marks.

1. Express the integral $\int_1^2 \sqrt{x}\, dx$ as limit of Riemann sums.   [Level:2] [CO1]

2. Let $f(x) = \begin{cases} 2x, & x \le 1 \\ 2, & x > 1 \end{cases}$. Then Evaluate   [Level:3] [CO1]

   (a) $\int_0^1 f(x)\, dx$

   (b) $\int_{-1}^1 f(x)\, dx$

   (c) $\int_1^{10} f(x)\, dx$

3. Evaluate $\int \left[\csc t - \sec t \tan t \right]\, dt$ [Level:3] [CO1]

4. Calculate $\int x e^{3x}\, dx$ [Level:3] [CO2]

5. Find the average value of the function $f(x) = e^x$ over the interval $[-1, \ln 5]$.   [Level:3] [CO2]

6. Evaluate $\displaystyle \lim_{(x,y) \to (-1,2)} \frac{xy}{x^2 + y^2}$.   [Level:2] [CO3]

7. If $f(\rho, \theta, \phi) = \rho^2 \cos \phi \sin \theta$, then find f_ρ, f_θ, f_ϕ.   [Level:3] [CO3]

8. Show that the mixed second-order partial derivatives of $f(x, y) = e^x \cos y$ are the same.   [Level:2] [CO4]

9. Find the eigenvalues of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & -7 \end{pmatrix}$.   [Level:2] [CO5]

10. Is matrix multiplication commutative? Verify the result for the given two matrices.   [Level:2] [CO5]

   $A = \begin{pmatrix} 1 & 4 \\ 5 & 10 \\ 8 & 12 \end{pmatrix}$ and $B = \begin{pmatrix} -4 & 6 & -3 \\ 1 & -3 & 2 \end{pmatrix}$

(Ceiling: 24 Marks)

Part B (Paragraph questions/Problem)

Answer *all* questions. Each question carries 6 marks.

11. Evaluate the integrals using appropriate substitutions: (a) $\int \frac{dx}{1+16x^2}$ (b)   [Level:3] [CO1]
$\int t\sqrt{7t^2 + 12}\, dt$

12. Using the method of partial fractions, calculate $\int \frac{11x+17}{2x^2+7x-4}\, dx$ [Level:3] [CO2]

13. Using substitution, compute (a) $\int_1^3 \frac{x+2}{\sqrt{x^2+4x+7}}\, dx$ (b) $\int_1^2 \frac{dx}{x^2-6x+9}$ [Level:3] [CO2]

14. Find the arc length of the curve $y = 3x^{3/2} - 1$ from $x = 0$ to $x = 1$ [Level:3] [CO2]

15. Let $f(x, y) = x + 3x^2y^2$, $x(t) = t^2$ and $y(t) = t^3$. Find   [Level:2] [CO3]

(a) $f(x(t), y(t))$

(b) $f(x(0), y(0))$

(c) $f(x(2), y(2))$

16. Describe the level surfaces of $f(x, y, z) = x^2 + y^2 + z^2$.   [Level:2] [CO3]

17. Find the eigenvalues and eigen vectors of $A = \begin{pmatrix} -1 & 2 \\ -7 & 8 \end{pmatrix}$.   [Level:2] [CO5]

18. Construct an orthogonal matrix from the eigenvectors of the given symmetric   [Level:3] [CO5]
matrix $\begin{pmatrix} 1 & 9 \\ 9 & 1 \end{pmatrix}$

(Ceiling: 36 Marks)

Part C (Essay questions)

Answer any *one* question. The question carries 10 marks.

19. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line   [Level:3] [CO2]
$y = -x$

20. Solve the linear system using Gaussian elimination.   [Level:3] [CO5]
$$2x_1 + 6x_2 + x_3 = 7$$
$$x_1 + 2x_2 - x_3 = -1$$
$$5x_1 + 7x_2 - 4x_3 = 9$$

(1 × 10 = 10 Marks)

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