23U401 (Pages: 2) Name : ....

Reg. No : ....

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2025

(CBCSS-UG)

(Regular/Supplementary/Improvement)

CC19U MTS4 B04 / CC20U MTS4 B04 - LINEAR ALGEBRA

(Mathematics - Core Course)

(2019 Admission onwards)

Time: 2.5 Hours Maximum: 80 Marks

Credit: 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Is the system of equations $4x - 2y = 1;\ 16x - 8y = 4$ is consistent.
2. Find the two different row echelon forms of $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$.
3. Determine whether the given matrices are elementary (a) $A = \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix}$, (b) $B = \begin{bmatrix} -5 & 1 \\ 1 & 0 \end{bmatrix}$, (c) $C = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$
4. Find A^{-1}, if $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$
5. What are the standard basis vector of $\mathbb{R}^n$?
6. Find the valus of λ for which $det(A) = 0$ when $A = \begin{bmatrix} \lambda - 2 & 1 \\ -5 & \lambda + 4 \end{bmatrix}$.
7. Show that $e_1 = (1,0,0), e_2 = (0,1,0)\ and\ e_3 = (0,0,1)$ are linearly independent in $\mathbb{R}^3$.
8. Define Row space and Column Space.
9. Define rank of a matrix.
10. If $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $w_1 = 2x_1 + x_2$ and $w_2 = 3x_1 + 4x_2$. Find $T(1,1), T(0,1)$ and $T(3,1)$.
11. Find the equation of the image of the line $y = 4x$ under the multiplication by matrix $A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$.
12. Find the eigen vector of $A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$ corresponding to $\lambda = 2$.
13. Define standard inner product space.
14. Define an orthonormal set.
15. Find the new coordinates of the point $(5, 2)$ by rotating coordinate system through an angle $\theta = \pi/3$

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer ***all*** questions. Each question carries 5 marks.

16. Is $(A+B)(A-B)=A^2-B^2$? Justify your answer.

17. If $p(x)=x^2-2x-3$ and $A=\begin{bmatrix}-1 & 2\\ 0 & 3\end{bmatrix}$ then find $p(A)$.

18. Show that if $W_1, W_2 \ldots W_n$ are subspaces of a vector space V, then the intersection of these subspaces is also a subspace of V.

19. If $S=\{v_1, v_2 \ldots v_n\}$ is a basis for a vector space V, prove that every element in V can be expressed as a linear combination of elements of S in a unique way.

20. Find a basis for and the dimention of the solution of the homogeneou system

$$\begin{aligned} x_1+x_2-x_3&=0\\ -2x_1-x_2+2x_3&=0\\ -x_1+x_3&=0 \end{aligned}$$

21. Consider the bases $B=\{u_1,u_2\}$ and $B'=\{u_1',u_2'\}$ where $u_1=(1,0), u_2=(0,1)$ $u_1'=(1,1),\ u_2'=(2,1)$ find the transition matrix from B to B'

22. Use matrix multiplication to find the reflection of $(2,-3,5)$ about xy- plane, xz plane and the yz plane

23. Find the eigen values of $A=\begin{bmatrix}4 & 0 & 1\\ -2 & 1 & 0\\ -2 & 0 & 1\end{bmatrix}$

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any ***two*** questions. Each question carries 10 marks.

24. Find the A^{-1} using inverse algorithm if $A=\begin{bmatrix}2 & -4 & 0 & 0\\ 1 & 2 & 12 & 0\\ 0 & 0 & 2 & 0\\ 0 & -1 & -4 & -5\end{bmatrix}$

25. (a) Prove that $\begin{vmatrix}a_1+b_1t & a_2+b_2t & a_3+b_3t\\ a_1t+b_1 & a_2t+b_2 & a_3t+b_3\\ c_1 & c_2 & c_3\end{vmatrix}=(1-t^2)\begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}$

    (b) If B is the matrix that results when a single row or single column of A is multiplied by a scalar k then prove that $det(B)=k.\,det(A)$

26. a) State and prove Dimension Theorem.

    b) Verify Dimension Theorem for $A=\begin{bmatrix}1 & -2 & 2 & 3 & -1\\ -3 & 6 & -1 & 1 & -7\\ 2 & -4 & 5 & 8 & -4\end{bmatrix}$

27. Find an orthogonal matrix P that diagonalizes $A=\begin{bmatrix}4 & 2 & 2\\ 2 & 4 & 2\\ 2 & 2 & 4\end{bmatrix}$

(2 × 10 = 20 Marks)

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