22U602 (Pages: 2) Name : ....

Reg. No : ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2025

(CBCSS-UG)

(Regular/Supplementary/Improvement)

CC19U MTS6 B10 / CC20U MTS6 B10 - REAL ANALYSIS

(Mathematics - Core Course)

(2019 Admission onwards)

Time: 2.5 Hours Maximum: 80 Marks

Credit: 5

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Discuss the contiuity of the signum function at $x = 0$.
2. State boundedness theorem for continuous functions.
3. Give an example to show that a continuous function does not necessarily have an absolute maximum even though it has an absolute minimum on a set.
4. Show that the equation $x = \cos x$ has a solution in the interval $[0, \pi/2]$.
5. State location of roots theorem.
6. State Weierstrass approximation theorem.
7. Define norm of the partition. Calculate the norm of the partition P=(1, 2.2, 4.1, 6.8, 8.7, 10) of $[1, 10]$.
8. If $f \in \mathfrak{R}[a, b]$, then prove that the value of the integral is uniquely determined.
9. Define null set. Give an example.
10. Discuss the uniform convergence of the sequence of functions $(f_n(x))$, where $f_n(x) = x^n$.
11. Evaluate the improper integral $\int_{-\infty}^{0} \cosh x dx$.
12. Examine the absolute convergence of $\int_1^\infty \frac{\cos x}{x^2+1} dx$.
13. Examine the convergence of $\int_1^5 \frac{dx}{\sqrt{x^4-1}}$.
14. Prove that Beta function is symmetric.
15. Express $\int_0^1 \frac{x^2}{\sqrt{(1-x^5)}} dx$ in terms of Beta function.

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer ***all*** questions. Each question carries 5 marks.

16. Every uniformy continuous function is Lipschitz function. True or false. Justify your answer.
17. Let f and g are in $\mathfrak{R}[a,b]$. Prove that $f+g \in \mathfrak{R}[a,b]$ and $\int_a^b (f+g) = \int_a^b f + \int_a^b g$.
18. State and prove squeeze theorem.
19. Prove that a sequence (f_n) of bounded functions on $A \subseteq \mathbb{R}$ converges uniformly on A to f if and only if $\|f_n - f\|_A \to 0$ as $n \to \infty$.
20. Prove Cauchy criterion for uniform convergence of sequence of functions.
21. If a sequence of continuous functions (f_n) converges uniformly to a function f on $A \subseteq \mathbb{R}$, then prove that f is continuous on A.
22. State and prove Weierstrass M-test.
23. Prove that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$.

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any ***two*** questions. Each question carries 10 marks.

24. Prove that if $f : [a,b] \to \mathbb{R}$ is a monotone function then $f \in \mathfrak{R}[a,b]$.
25. State and prove first and second forms of fundamental theorem.
26. Show that even though the improper integral $\int_{-1}^{5} \frac{dx}{(x-1)^3}$ does not converge, its Cauchy principal value exists.
27. Establish the relation between Beta and Gamma function.

(2 × 10 = 20 Marks)

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