22U603 (Pages: 2) Name : ....

Reg. No : ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2025

(CBCSS-UG)

(Regular/Supplementary/Improvement)

CC19U MTS6 B11 / CC20U MTS6 B11 - COMPLEX ANALYSIS

(Mathematics - Core Course)

(2019 Admission onwards)

Time: 2.5 Hours Maximum: 80 Marks

Credit: 5

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Evaluate $\lim_{z\to\infty}\left(\frac{z^2+iz-2}{(1+2i)z^2}\right)$

2. Show that $f(z)=\frac{z^2+1}{z+i}$ is discontinuous at $z_0=-i$.

3. If $f(z)=\frac{3e^{2z}-ie^{-z}}{z^3-1+i}$, find the derivative $f'(z)$.

4. Prove that $|f'(z)|^2=u_x^2+v_x^2=u_y^2+v_y^2$

5. Show that the function $f(z)=\frac{x}{x^2+y^2}-i\frac{y}{x^2+y^2}$ is analytic in any domain that does not contain origin.

6. Write the principal value of the logarithm, $\operatorname{Ln}\left[(1+\sqrt{3}i)^5\right]$ in the form $a+ib$.

7. Show that $\cosh(iz)=\cos z$.

8. Evaluate $\oint_C(x^2-y^2)ds$ where C is the circle defined by $x=5\cos t, y=5\sin t; 0\leq t\leq 2\pi$.

9. State Cauchy-Goursat theorem for multiply connected domains.

10. Evaluate $\int_0^{3+i} z^2dz$

11. Using Cauchy's integral formula evaluate $\oint_C\frac{e^z}{z-\pi i}dz$ where C is the circle $|z|=4$.

12. Show that the sequence $\{z_n\}=\left\{\frac{3+ni}{n+2ni}\right\}$ converges to a complex number L by computing $\lim_{n\to\infty}\operatorname{Re}(z_n)$ and $\lim_{n\to\infty}\operatorname{Im}(z_n)$.

13. Expand $f(z)=\frac{1}{z}$ in a Taylor series with center $z_0=1$. Give the radius of convergence R.

14. Determine the zeros and their order for the function $f(z)=z^4+z^2$

15. Let $f(z) = \dfrac{2}{(z-1)(z+4)}$, find $\text{Res}(f(z), 1)$.

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer ***all*** questions. Each question carries 5 marks.

16. Show that the function $f(z) = 2x^2 + y + i(y^2 - x)$ is not analytic at any point, but is differentiable along the line $y = 2x$.

17. Show that $|\sin z|^2 = \sin^2 x + \sinh^2 y$.

18. Evaluate $\int_C (2\bar{z} - z)dz$ where C is given by $x = -t, y = t^2 + 2;\ 0 \le t \le 2$.

19. Evaluate $\oint_C \dfrac{z+2}{z^2(z-1-i)}dz$ where C is the circle $|z| = 1$.

20. State and prove Morera's theorem.

21. Determine whether the geometric series $\sum_{k=0}^{\infty} 3\left(\dfrac{2}{1+2i}\right)^k$ convergent or divergent. If convergent, find its sum.

22. Expand $f(z) = \dfrac{8z+1}{z(1-z)}$ in a Laurent series valid for $0 < |z| < 1$

23. Evaluate $\int_0^{2\pi} \dfrac{\cos\theta}{3 + \sin\theta}d\theta$

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any ***two*** questions. Each question carries 10 marks.

24. Show that the function $u(x, y) = 4xy^3 - 4x^3y + x$ is harmonic in the entire complex plane. Also find the harmonic conjugate function v and find analytic function $f(z) = u + iv$ satisfying the condition $f(1+i) = 5 + 4i$.

25. State and prove the bounding theorem. Use it to find an upper bound for the absolute value of the integral $\int_C \dfrac{e^z}{z+1}dz$ where C is the circle $|z| = 4$.

26. Show that the power series $\sum_{k=1}^{\infty} \dfrac{(z-i)^k}{k2^k}$ is not absolutely convergent on its circle of convergence. Determine at least one point on the circle of convergence at which the power series is convergent.

27. State and prove residue theorem. Using residue theorem evaluate $\oint_C \dfrac{1}{z^2+4z+13}dz$ where C is the circle $|z - 3i| = 3$.

(2 × 10 = 20 Marks)

****