22U604 (Pages: 2) Name : ....

Reg. No : ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2025

(CBCSS-UG)

(Regular/Supplementary/Improvement)

CC19U MTS6 B12 / CC20U MTS6 B12 - CALCULUS OF MULTIVARIABLE

(Mathematics - Core Course)

(2019 Admission onwards)

Time: 2.5 Hours Maximum: 80 Marks

Credit: 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Find and sketch the domain of the function $f(x,y)=\frac{ln(x+y+1)}{y-x}$.
2. Determine where the function $f(x,y)=ln(2x-y)$ is continuous.
3. Find the first partial derivatives of the function $f(x,y)=x\sqrt{y}$.
4. Find f_x if $f(x,y,z)=x^2y+y^2z+xz$.
5. Let $w=x^2y+y^2z^3$, where $x=r\cos s, y=r\sin s, z=re^s$. Find the value of $\frac{\partial w}{\partial s}$ when $r=1, s=0$.
6. Suppose f is a differentiable at the point (x,y). Then prove that the maximum value of $D_u f(x,y)$ is $|\bigtriangledown f(x,y)|$, and this occurs when u has the same direction as $\bigtriangledown f(x,y)$.
7. Evaluate $\int_0^1\int_0^2(x+2y)dydx$
8. Evaluate $\int\int_R xydA$ where $R=\left\{(x,y):-1\le x\le 2, -x^2\le y\le 1+x^2\right\}$.
9. Using polar coordinates evaluate $\int\int_R 3ydA$, where R is the disk of radius 2 centered at the origin.
10. Evaluate $\int_1^4\int_0^1\int_0^2 z^2dxdydz$
11. Find the jacobian of the transformation T defined by $x=e^u\cos 2v, y=e^u\sin 2v$.
12. Find the gradient vector field of $f(x,y)=x^2y-y^3$.
13. Find the curl of $F(x,y,z)=-y\hat{i}+x\hat{j}$.
14. Identify the surface represented by $r(u,v)=2\cos u\hat{i}+2\sin u\hat{j}+v\hat{k}$ with parameter domain $D=\{(u,v)|0\le u\le 2\pi, 0\le v\le 3\}$.
15. State Stockes' Theorem.

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer ***all*** questions. Each question carries 5 marks.

16. Let $z = 2x^2 - xy$.
(i) Find the differential dz.
(ii)Compute the value of dz if (x, y)changes from (1,1) to (0.98,1.03).

17. Find equations of the tangent plane and normal line to the ellipsoid with equation $4x^2 + y^2 + 4z^2 = 16$ at the point $(1, 2, \sqrt{2})$.

18. Find the moments of inertia with respect to the x- axis of a thin homogeneous disk of mass m and radius a centered at the origin with $\rho(x, y) = \dfrac{m}{\pi a^2}$.

19. Find the area of that part of the plane$y + z = 2$ inside the cylinder $x^2 + z^2 = 1$.

20. Let $F(x, y) = -\frac{1}{8}(x - y)\hat{i} - \frac{1}{8}(x + y)\hat{j}$ represents a force field. Find the work done on a particle that moves along the quarter-circle of radius 1 centered at the origin in a counterclockwise direction from $(1, 0)$ to $(0, 1)$.

21. Let $F(x, y) = (2xy^2 + 2y)\hat{i} + (2x^2y + 2x)\hat{j}$.
(a) Show that F is conservative, and find a potential function f such that $F = \bigtriangledown f$.
(b) If F is a force field, find the work done by F in moving a particle along any path from $(-1, 1)$ to $(1, 2)$.

22. Using Green's theorem, evaluate $\oint_C (y^2 + \tan x)dx + (x^3 + 2xy + \sqrt{y})dy$, where C is the circle $x^2 + y^2 = 4$ and is oriented in a positive direction.

23. Evaluate $\int\int_S x dS$, where S is the part of the plane $2x + 3y + z = 6$ that lies in the first octant.

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any ***two*** questions. Each question carries 10 marks.

24. Find the absolute maximum and the absolute minimum values of the function $f(x, y) = xy - x^2$ on the region bounded by the parabola $y = x^2$ and the line $y = 4$.

25. Find the maximum and minimum values of the function $f(x, y) = x^2 - 2y$ subject to $x^2 + y^2 = 9$.

26. Find the center of mass of the solid$T$of uniform density bounded by the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2 + y^2 + z^2 = z$.

27. Verify Divergence Theorem for the vector field $F(x, y, z) = 2xy\hat{i} - y^2\hat{j} + 3yz\hat{k}$ and the region T bounded by the planes $x = 0, x = 2, y = 0, y = 2, z = 0, z = 2$.

(2 × 10 = 20 Marks)

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