22U605 (Pages: 2) Name : ....

Reg. No : ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2025

(CBCSS-UG)

(Regular/Supplementary/Improvement)

CC19U MTS6 B13 / CC20U MTS6 B13 - DIFFERENTIAL EQUATIONS

(Mathematics - Core Course)

(2019 Admission onwards)

Time: 2.5 Hours Maximum: 80 Marks

Credit: 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Solve the initial value problem $\frac{dy}{dt} = -2y + 10;\ y(0) = 0$
2. Determine the order of the given differential equation $\frac{d^4y}{dt^4} + \frac{d^3y}{dt^3} + 2y = \sin t$ also state whether the equation is linear or nonlinear.
3. Determine the value of r for which the differential equation $t^2y'' - 4ty' + 4y = 0$ has solutions of the form $y = t^r$.
4. Solve the equation $\frac{dy}{dx} = \frac{x}{y}$
5. Find an interval in which the initial value problem $(t-3)y' + (\ln t)y = 2t$; $y(1) = 2$ has a unique solution.
6. State the condition under which the differential equation $M(x, y) + N(x, y)y' = 0$ is exact.
7. What do you mean by an integrating factor of a differential equation?
8. Find the longest interval in which the solution of the initial value problem $(t^2 - 3t)y'' + ty' - (t+3)y = 0, \quad y(1) = 2, \quad y'(1) = 1$ is certain to exist.
9. What is the radius of convergence of the Taylor series for $(1 + x^2)^{-1}$ about $x = 0$.
10. Find the Laplace transform of t
11. Find the inverse Laplace transform of $\frac{s-a}{(s-a)^2 + b^2}$
12. Write the expression for $\mathcal{L}(f'(t))$ and $\mathcal{L}(f''t))$
13. Prove that convolution of two functions f and g are commutative.
14. Find $\sin t * \sin t$

15. Prove that the product of an even function and an odd function is odd.

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer ***all*** questions. Each question carries 5 marks.

16. Find the solution of initial value problem $4y'' - 8y' + 3y = 0, \quad y(0) = 2, \quad y'(0) = \frac{1}{2}$

17. Find the Wronskian of two solutions of the given differential equation $2t^2y'' + 3ty' - y = 0$ by using Abel's formula.

18. Solve the initial value problem $y'' + 4y' + 4y = 0, \quad y(-1) = 2, \quad y'(-1) = 1$

19. Find a particular solution of $y'' - 3y' - 4y = 2\sin t$

20. Use the method of variation of parameters find the general solution of the differential equation $y'' + y = e^t/t^3$

21. Write the eigen values and eigen functions of the boundary value problem $y'' + \lambda y = 0; y(0) = 0, y(L) = 0$, when $\lambda > 0$

22. Find the Fourier series of $f(x) = x, -1 \le x < 1; f(x+2) = f(x)$.

23. Find the solution of the heat conduction problem
$$100u_{xx} = u_t; 0 < x < 1, t > 0$$
$$u(0,t) = 0, u(1,t) = 0, t > 0$$
$$u(x,0) = 2\sin(2\pi x) - \sin(5\pi x), 0 \le x \le 1$$

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any ***two*** questions. Each question carries 10 marks.

24. Find the general solution of $y'' + 2y' = 3 + 4\sin(2t)$

25. Solve the initial value problem $y'' - y' - 2y = 0$ that satisfies the initial condition $y(0) = 1, y'(0) = 0$. using Laplace transform

26. Find the solution of the initial value problem $2y'' + y' + 2y = \delta(t-5) \;\; y(0) = 0, y'(0) = 0$

27. Let $f(x) = \begin{cases} x & -\pi \le x < 0, \\ 0, & 0 \le x < \pi, \end{cases}$ and suppose that $f(x+2\pi) = f(x)$. Find the Fourier series for f.

(2 × 10 = 20 Marks)

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