24P201 (Pages: 2) Name: ....

Reg.No: ....

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2025

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MTH2 C06 - ALGEBRA - II

(Mathematics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Show that $\mathbb{Z}_5[x]/\langle x^3+3x+2\rangle$ is a field.
2. Prove that $\mathbb{R}(i) \cong \mathbb{C}$
3. Prove that doubling the cube is impossible.
4. Prove that finite extension of a finite field is finite.
5. Prove that any two algebraic closures of a field F are isomorphic.
6. Find the splitting field of $\{x^4-5x^2+6\}$.
7. Find $\Phi_5(x)$ over $\mathbb{Q}$.
8. Prove that $\mathbb{Q}(\sqrt{2})$ is an extension of $\mathbb{Q}$ by radicals.

(8 × 1 = 8 Weightage)

Part B

Answer any ***two*** questions each unit. Each question carries 2 weightage.

UNIT - I

9. If E is finite extension field of a field F, and K is a finite extension field of E, then prove that K is a finite extension of F .and $[K:F]=[K:E][E:F]$.
10. Find a basis and dimension of for $\mathbb{Q}(\sqrt{2},\sqrt{3})$ over $\mathbb{Q}$.
11. Let E be an extension field of F, then prove that $\overline{F}_E=\{\alpha\in E:\alpha \text{ is algebraic over } F\}$ is a subfield of E.

UNIT - II

12. A finite field $GF(p^n)$ of p^n elements exist for every prime power p^n.
13. Find the splitting field of x^3-2 over $\mathbb{Q}$.
14. If K is a finite extension of E and E is a finite extension of F, that is $E\le F\le K$, then K is separable over F if and only if K is separable over E and E is separable over F.

UNIT - III

15. Prove that a finite seperable extension of a field is a simple extension.

16. If K splitting field of x^4+1 over $\mathbb{Q}$, prove that $G(K/\mathbb{Q})$ is isomorphic to Klein 4-group.

17. Find $\Phi_8(x)$ over $\mathbb{Z}_2$.

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. Let R be a commutative ring with unity. Then show that M is a maximal ideal of R if and only if R/M is a field.

19. State and Prove Kroneckers Theorem.

20. State and Prove The Conjugation Isomorphism theorem.

21. Prove that the Galois group of pth cyclotomic extension of $\mathbb{Q}$ for a prime p is cyclic of order $p-1$.

(2 × 5 = 10 Weightage)

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