24P203 (Pages: 2) Name: ....

Reg.No: ....

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2025

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MTH2 C08 - TOPOLOGY

(Mathematics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Define closure of a subset of a topological space. If A is a closed set prove that $\bar{A} = A$.
2. Let $(X, \mathcal{T})$ be a topological space $A \subset X$. Prove that $int(A)$ is the union of all open sets contained in A.
3. Define projection functions. Prove that projection functions are not closed.
4. Define weak topology determined by the family of functions $\{f_i : i \in I\}$.
5. Prove that every open surjective map is a quotient map.
6. Prove that a space X is locally connected at a point $x \in X$, if and only if for every nieghbourhood N of x the component of N containing x is a a nieghbourhood of x .
7. Prove that a compact subset in a Hausdorff space is closed.
8. State Urysohn's lemma

(8 × 1 = 8 Weightage)

Part B

Answer any ***two*** questions each unit. Each question carries 2 weightage.

UNIT - I

9. Let $f : X \to Y$ be a function where X, Y are metric spaces and let $x_0 \in X$. Then prove that f is continuous at x_0 iff for every open set V in Y containing $f(x_0)$, there exists an open set U in X containing x_0 such that $f(U) \subset V$.
10. Let X be a set and let $\mathcal{T}$ consists of all those subsets of X whose complements are countable together with the empty set. Show that $\mathcal{T}$ is a topology.
11. Define hereditary property. Prove that metrisability is a hereditary property.

UNIT - II

12. Define second countable space. Is every second countable space first countable? Justify.
13. Prove that topological product of any finite number of connected spaces is connected.

14. Define path connected space. Prove that every path connected space is connected.

UNIT - III

15. Define Hausdorff space. Prove that in a Hausdroff space, limits of sequences are unique.

16. Define regular space and show that regularity is a hereditary property.

17. Prove that every regular Lindeloff space is normal.

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. Prove that the product topology on $\mathcal{R}^n$ coincides with the usual topology on it.

19. (a) Prove that every continuous real valued function on a compact space is bounded and attains its extrema.

    (a) State and prove Lebesque covering lemma.

20. (a) Prove that a subset of $\mathcal{R}$ is connected iff it is an interval.

    (b) Prove that connectedness is preserved under a continuous surjection.

21. Let A be a closed subset of a normal space X and suppose $f : A \to (-1,1)$ is a continuous function. Then prove that there exists a continuous function $F : X \to (-1,1)$ such that $F(x) = f(x)$ for all $x \in A$.

(2 × 5 = 10 Weightage)

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