**24P204** (Pages: 2) Name: ...............................

Reg.No: ...............................

**SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2025**

(CBCSS - PG)

(Regular/Supplementary/Improvement)

**CC19P MTH2 C09 - ODE & CALCULUS OF VARIATIONS**

(Mathematics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

**Part A**

Answer ***all*** questions. Each question carries 1 weightage.

1. Find a power series solution of the differential equation $y' + y = 1$.
2. Define ordinary point of a differential equation $y'' + p(x)y' + Q(x)y = 0$ and give an example of differential equation having ordinary point $x = 1$.
3. For the Legendre polynomial $P_n(x)$, prove that $P_n(1) = 1$.
4. Describe the phase portrait of the system $\frac{dx}{dt} = 0, \ \frac{dy}{dt} = 0$.
5. Determine the nature and stability properties of the critical point $(0,0)$ of the linear autonomous system $\frac{dx}{dt} = 4x - 2y, \ \frac{dy}{dt} = 5x + 2y$.
6. Determine whether the function $f(x,y) = -x^2 - 4xy - 5y^2$ is positive definite, negative definite or neither.
7. If $q(x) < 0$ and $u(x)$ is a nontrivial solution of $u'' + q(x)u = 0$, then prove that $u(x)$ has atmost one zero.
8. Show that $f(x,y) = xy^2$ satisfies a Lipschitz condition on any rectangle $a \leq x \leq b$ and $c \leq y \leq d$.

**(8 × 1 = 8 Weightage)**

**Part B**

Answer any ***two*** questions each unit. Each question carries 2 weightage.

**UNIT - I**

9. Calculate two independent Frobenius series solutions of the differential equations $4xy'' + 2y' + y = 0$.
10. Determine the nature of the point $x = \infty$ for the Confluent hypergeometric equation $xy'' + (c - x)y' - ay = 0$.
11. Find the first three terms of the Legender series of $f(x) = e^x$.

**UNIT - II**

12. Find the general solution of $\frac{dx}{dt} = -4x - y, \ \frac{dy}{dt} = x - 2y$.

13. Describe about different types of critical points.

14. Verify that $(0, 0)$ is a simple critical point for the system $\frac{dx}{dt} = -x - y - 3x^2y, \ \frac{dy}{dt} = -2x - 4y + y\sin x$ and determine its nature and stability properties.

**UNIT - III**

15. State and prove Sturm comparison theorem.

16. Find the exact solution of the initial value problem $y' = y^2$ with $y(0) = 1$. Apply Picard's method to calculate $y_1(x), y_2(x), y_3(x)$ (start with $y_0(x) = 1$) and compare these results with exact solution.

17. Prove that the plane curve of fixed perimeter and maximum area is the circle.

**(6 × 2 = 12 Weightage)**

**Part C**

Answer any ***two*** questions. Each question carries 5 weightage.

18. (i) Determine the general solution of the hypergeometric equation
$x(1-x)y'' + [c - (a+b+1)x]y' - aby = 0$ near $x = 0$.
(ii) Verify that $\cos x = x[\lim_{a\to\infty} F(a, a, \frac{1}{2}, \frac{-x^2}{4a^2})]$.

19. (i) Derive Bessel's function of first kind.
(ii) Show that between any two positive zeros of $J_0(x)$, there is a zero of $J_1(x)$.

20. State and prove the orthogonality property of Bessel functions.

21. Derive Euler's equation for an extremal and find the stationary function of $\int_{x_1}^{x_2} [xy' - (y')^2]dx$ which is determined by the boundary conditions $y(0) = 0$ and $y(4) = 3$.

**(2 × 5 = 10 Weightage)**

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