17P201 (Pages: 2) Name…………………

Reg.No………………..

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, MAY 2018

(Regular/Supplementary/Improvement)

(CUCSS - PG)

CC15P MT2 C06 / CC17P MT2 C07 – ALGEBRA II

(Mathematics)

(2015Admission onwards)

Time: Three Hours Maximum: 36 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Define prime fields.
2. State Kronecker's Theorem.
3. Prove that the field $\mathbb{C}$ of complex numbers is an algebraically closed field.
4. If $\propto$ and β are constructable real numbers, prove that $\propto \beta$ is constructable.
5. Find the number of primitive 10^{th} roots of unity in GF(23).
6. Find all conjugates in $\mathbb{C}$ of $3+\sqrt{2}$ over Q.
7. Let σ be the automorphism of Q(π) that maps π onto $-\pi$. Describe the fixed field of σ
8. Find the degree over Q of the splitting field over Q of x^3-1 in Q[x].
9. Show that $Q[\sqrt{2}, \sqrt{3}]$ is separable over Q.
10. Give an example of two finite normal extensions K_1 and K_2 of the same field F such that K_1 and K_2 are not isomorphic fields but $G(K_1/F) \simeq G(K_2/F)$.
11. Define n^{th} cyclotomic polynomial over the field F.
12. Show that the polynomial $(x^2 - 2)(x^2 - 3)$ is solvable by radicals over Q.
13. If E is a finite extension of F, then show that {E:F} divides [E:F].
14. Prove that a finite extension field E of a field F is an algebraic extension of F.

(14 × 1 = 14 Weightage)

Part B

Answer any ***seven*** questions. Each question carries 2 weightage.

15. Show that there exists a finite field of 9 elements.
16. Let E be a finite extension of a field F, and let $p(x) \in F[x]$ be irreducible over F and have degree that is not a divisor of $[E{:}F]$. Show that $p(x)$ has no zeros in E.
17. Show that the regular 9 – gon is not constructable.
18. If F is any finite field, prove that there exist an irreducible polynomial in F[x] for every positive integer n.

19. If $E \leq \bar{F}$ is a splitting field over F, Prove that every irreducible polynomial in F[x] having a zero in E splits in E.
20. Show that if [E: F] = 2, then E is a splitting field over F.
21. Let K be a finite extension of degree n of a finite field F of p^r elements. Prove that G(K/F) is cyclic of order n and is generated by σ_{p^r}, where for $\alpha \in K$, $\sigma_{p^r}(\alpha) = \alpha^{p^r}$.
22. Prove that the Galois group of p^{th} cyclotomic extension of Q for a prime p is cyclic of order p-1.
23. Let F be a field of characteristic 0, and let $a \in F$ .If K is the splitting field of $x^n - a$ over F, prove that G(K/F) is a solvable group.
24. Prove that a finite extension E of a finite field F is a simple extension of F.

(7 × 2 = 14 Weightage)

Part C

Answer any ***two*** questions. Each question carries 4 weightage.

25. Let R be a commutative ring with unity. Prove that M is a maximal ideal of R if and only if R/M is a field.
26. State and prove Conjugation isomorphism Theorem.
27. Let E be a finite separable extension of a field F. Prove that there exists $\alpha \in E$ such that $E = F(\alpha)$.
28. State and prove Isomorphism Extension Theorem.

(2 × 4 = 8 Weightage)

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