17P206 (Pages: 2) Name…………………………

Reg. No……………………..

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, MAY 2018

(CUCSS - PG)

(Mathematics)

CC17P MT2 C10 - ODE AND CALCULUS OF VARIATIONS

(2017 Admissions: Regular)

Time: Three Hours Maximum: 36 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Define Interval of convergence and find Interval of convergence for the series $\sum_{n=0}^{\infty} x^n$
2. Locate and classify singular points of $x^3(x-1)y'' - 2(x-1)y' + 3xy = 0$
3. Find the indicial equation and its roots for $4xy'' + 2y' + y = 0$
4. Show that $e^x = \lim_{b\to\infty} F(a, b, a, \frac{x}{b})$
5. Find the first two terms of the Legendre series for f(x)=e^x
6. Show that $\left(n+\frac{1}{2}\right)! = \frac{(2n+1)!\sqrt{\pi}}{2^{n+1}\ n!}$
7. State orthogonal property of Bessel function
8. State Bessel expansion theorem
9. Find critical points and phase portrait of $\begin{cases} \frac{dx}{dt} = -x \\ \frac{dy}{dt} = -y \end{cases}$
10. Show that the function E(x,y) = a$x^2 + bxy + cy^2$ is positive definite if and only if a > 0 and $b^2 - 4ac < 0$
11. Explain Picard's method of successive approximation
12. Show that f(x,y) = $y^{1/2}$ does not satisfy Lipschitz condition on the rectangle |x|≤ 1 and $0 \leq y \leq 1$.
13. Find the Extremal of the integral $\int_{x_1}^{x_2} (y^2 - y'^2)$ dx.
14. Show that every non trivial solution of $y'' + (sin^2 x + 1)y = 0$ has an infinite number of positive zeros **(14 x 1 = 14 Weightage)**

Part B

Answer any ***seven*** questions. Each question carries 2 weightage.

15. Find the general solution of Legendre's equation in terms of power series

16. Find the general solution of (1-e^x)$y'' + \frac{1}{2}y' + e^x y = 0$ near the singular point x = 0 by changing the independent variable to t = e^x
17. Show that x = ∞ is an irregular singular point for the confluent hyper geometric equation
18. Show that $\frac{d}{dx}\{x^{-p}J_p(x)\} = -x^{-p}J_{p+1}(x)$
19. Determine the nature and stability property of the critical point (0, 0) for $\begin{cases} \frac{dx}{dt} = -3x + 4y \\ \frac{dy}{dt} = -2x + 3y \end{cases}$
20. Define Liapunov function and prove that the critical point (0,0) is stable if there exist a Liapunove function E(x,y) for the system $\begin{cases} \frac{dx}{dt} = F(x,y) \\ \frac{dy}{dt} = G(x,y) \end{cases}$
21. Verify that (0,0) is a simple critical point of the system $\begin{cases} \frac{dx}{dt} = \quad x + y - 2xy \\ \frac{dy}{dt} = -2x + y + 3y^2 \end{cases}$ and determine the nature of the critical point
22. Solve the initial value problem using Picard's method $\begin{cases} \frac{dy}{dx} = z,\ y(0) = 1 \\ \frac{dz}{dx} = -y, z(0) = 0 \end{cases}$
23. Find the curve of fixed length L that joins the point (0,0) and (0,1) lies above the X - axis and encloses the maximum area between itself and the X- axis
24. State and prove Sturm's separation theorem

(7 x 2 = 14 Weightage)

Part C

Answer any ***two*** questions. Each question carries 4 weightage.

25. Solve $4x^2y'' - 8x^2y' + (4x^2 + 1)y = 0$
26. State and prove orthogonal property of Legendre polynomials
27. Find the general solution of $\begin{cases} \frac{dx}{dt} = 5x + 4y \\ \frac{dy}{dt} = -x + y \end{cases}$
28. Let y(x) and z(x) be non trivial solutions of $y'' + q(x)y = 0$ and $z'' + r(x)z = 0$, where q(x) and r(x) be positive functions such that q(x) > r(x) then show that y(x) vanishes at least once between every two successive zeroes of z(x)

(2 x 4 = 8 Weightage)

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