17P208 (Pages: 3) Name…………..…………..

Reg. No……………………

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, MAY 2018

(CUCSS - PG)

(Mathematics)

CC17P MT2 C11 - OPERATIONS RESEARCH

(2017 Admissions Regular)

Time: Three Hours Maximum: 36 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Define a convex function. Show that sum of two convex functions is a convex function.
2. Show that if the convex function has a relative minimum at X_0, then it is also a global minimum.
3. Explain the terms : basic solution and feasible solution
4. Show that the dual of the dual is the primal.
5. Explain the transportation matrix of a transportation problem
6. Explain degeneracy in transportation problem with an example.
7. What is sensitivity analysis. Discuss how changes in cost coefficients (c_j) affects the original LPP.
8. Find the saddle point, if it exists, for the pay off matrix $\begin{bmatrix} 2 & 2 & 2 & 2 \\ 1 & 2 & 3 & 4 \end{bmatrix}$
9. Define the terms in game theory: pay off, mixed strategy, saddle point and strategic saddle point
10. Define the terms: tree, centre of a tree, arborescence.
11. Write the dual of the following LPP:

    Maximize $f = x_1 - x_2 + 2x_3$, subject to

    $$x_1 - x_2 + x_3 = 4$$
    $$x_1 + x_2 - x_3 \geq 3,$$
    $$2x_1 - 2x_2 + 3x_3 \leq 15$$
    $$x_1, x_2 \geq 0, \quad x_3 \text{ unrestricted in sign.}$$
12. Discuss the unbalanced transportation problem
13. Define a parametric linear programming problem.
14. Define a flow in a graph.

(14 x 1 = 14 Weightage)

PART B

Answer any ***seven*** questions. Each question carries 2 weightage.

15. Show that a vertex of set of all feasible solutions of a LPP is a basic feasible solution.
16. Show that the optimum value of the primal if it exits, equals the optimum value of the dual.
17. Find an optimal solution of the transportation problem for minimum cost with the cost coefficients, demands and supplies as given in the table

	D1	D2	D3	D4	
O1	1	2	-2	3	70
O2	2	4	0	1	38
O3	1	2	-2	5	32
	40	28	30	42	

18. Define triangular basis. Show that the transportation problem has a triangular basis.
19. Show that the maximum flow in a graph is equal to the minimum of the capacities of all possible cuts in it.
20. Using branch and bound method

Maximize $3x_1 + 4x_2$; subject to

$2x_1 + 4x_2 \le 13$

$-2x_1 + x_2 \le 2$

$2x_1 + 2x_2 \ge 1$

$6x_1 - 4x_2 \le 15$, where x_1 , x_2 are non negative integers.

21. For the problem,

Maximize $f = x_1 - x_2 + 2x_3$, subject to

$x_1 - x_2 + x_3 \le 4$

$x_1 + x_2 - x_3 \le 3$

$2x_1 - 2x_2 + 3x_3 \le 15$

$x_1, x_2, x_3 \ge 0$, assuming that x_4, x_5, x_6 as slack variables, the optimal table is as follows.

Basis	Values	x_1	x_2	x_3	x_4	x_5	x_6
x_3	21	4		1		2	1
x_4	7	2			1	1	0
x_2	24	5	1			3	1
$-f$	18	2				1	1

Carry out the sensitivity analysis for the following changes and give the corresponding optimal solution

i) Coefficient of x_1 in the objective function changes to 2

ii) First constraint is deleted

22. State the general integer linear programming problem. Show that optimal solutions of an ILPP exists if optimal solution of related LPP exists. Give a relation between optimal solution of ILPP and related LPP.
23. Discuss the problem of maximum flow in a network and develop an algorithm to solve it.
24. State and prove the fundamental theorem of rectangular games.

(7 x 2 = 14 Weightage)

PART C

Answer any ***two*** questions. Each question carries 4 weightage.

25. Solve the linear programming problem by solving its dual

Maximize $y_1 + y_2 + y_3$ subject to

$2y_1 + y_2 + 2y_3 \le 2$,

$4y_1 + 2y_2 + y_3 \le 2$,

$y_1, y_2, y_3 \ge 0$

26. a) Describe the method of finding spanning tree of minimum length.

b) Describe the algorithm of finding minimum path in a graph with all arc lengths non negative.

27. Maximize $x_1 + x_2$; subject to constraints

$7x_1 - 6x_2 \le 5$

$6x_1 + 3x_2 \ge 7$

$-3x_1 + 8x_2 \le 6$ x_1 , x_2 are non negative integers.

28. Explain the notion of dominance in game theory. Use the notion of dominance and hence solve the game with payoff matrix $\begin{bmatrix} 0 & 5 & -4 \\ 3 & 9 & -6 \\ 3 & -1 & 2 \end{bmatrix}$

(2 x 4 = 8 Weightage)

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