17P203 (Pages: 2) Name……………………….

Reg. No…………………..

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, MAY 2018

(CUCSS - PG)

(Mathematics)

CC17P MT2 C08 – REAL ANALYSIS - II

(2017 Admission: Regular)

Time: Three Hours Maximum: 36 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Show that the set of rational numbers has outer measure zero.
2. If $m^*(E) = 0$, then show that E is measurable.
3. Show that continuous functions are measurable.
4. If f is a measurable function and if B is a Borel set, then show that $f^{-1}(B)$ is a measurable set.
5. State Fatou's Lemma.
6. Show that if f is integrable, then f is finite valued a.e.
7. Give an example where $D^+(f+g) \neq D^+(f) + D^+(g)$.
8. Show that if f is of bounded variation on $[a, b]$, then f is bounded on $[a, b]$.
9. Show that the Lebesgue set of an integrable function f contains any point at which f is continuous.
10. What do you mean by a complete measure?
11. Define signed measure.
12. If v_1, v_2 and μ are measures and $v_1 \perp \mu$, $v_2 \perp \mu$, then show that $(v_1 + v_2) \perp \mu$.
13. Is it true that every continuous function absolutely continuous? Justify.
14. State Riesz representation theorem for *C(I)*.

(14 × 1 = 14 Weightage)

PART B

Answer any ***seven*** questions. Each question carries 2 weightage.

15. Show that Lebesgue outer measure has the property of countable sub additivity.
16. Show that every Borel set is measurable.
17. Prove that the characteristic function χ_A of the set *A* is measurable iff *A* is measurable.
18. If f is an integrable function and if *A* and *B* are disjoint measurable sets, then prove that $\int_{A\cup B} f\, dx = \int_A f\, dx + \int_B f\, dx$.

19. Show that a function $f \in BV[a,b]$, if and only if f is the difference of two finite-valued monotone increasing functions on $[a,b]$, where a, b are finite.

20. If f is Lebesgue integrable over (a,b) and if $\int_a^x f dt = 0, \forall x \in (a,b)$, then show that $f = 0$ a.e. in (a,b).

21. Show that if μ is not complete, then f is measurable and $f = g$ a.e. do not imply that g is measurable.

22. If μ is a measure, $\int f d\mu$ exists and $\nu(E) = \int_E f d\mu$, then prove that ν is absolutely continuous w.r.t. μ.

23. If f is absolutely continuous on $[a,b]$, where a, b are finite, then show that $f \in BV[a,b]$.

24. Show that every bounded linear functional F on *C(I)* can be written as $F = F^+ - F^-$ where F^+, F^- are positive linear functionals.

(7 × 2 = 14 Weightage)

PART C

Answer any ***two*** questions. Each question carries 4 weightage.

25. (a). Show that the class $\mathcal{M}$ of Lebesgue measurable sets is a $\sigma -$ algebra.

    (b). Show that every interval is measurable.

26. (a). State and prove Lebesgue's dominated convergence theorem.

    (b). If f is integrable, then prove that $\left|\int f\, dx\right| \leq \int |f|\, dx$.

27. State and prove Lebesgue's differentiation theorem.

28. State and prove Radon - Nikodym Theorem.

(2 × 4 = 8 Weightage)

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