17P202 (Pages: 2) Name……………………….

Reg. No…………………..

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, MAY 2018

(Supplementary/Improvement)

(CUCSS - PG)

CC15P MT2 C07 – REAL ANALYSIS II

(Mathematics)

(2015, 2016 Admissions)

Time: Three Hours Maximum: 36 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Prove or disprove : If f and g are real measurable function defined on [a,b], where a < b, then fg is measurable.
2. Show that there is a strictly increasing singular function on $[0,1]$
3. Show that BA is linear if $A\ and\ B$ are linear transformations.
4. Find the derivative of a linear transformation $A: \mathbb{R}^n \to \mathbb{R}^m$at each point of $\mathbb{R}^n$
5. Prove that range of liner operator is a subspace.
6. Does there exists an algebra which is not a $\sigma - Algebra$ Justify your claim
7. Prove that the outer measure of a finite set is zero.
8. Show that product of two absolutely continuous functions is absolutely continuous.
9. Prove that to every $A \in L(\mathbb{R}^n, \mathbb{R}^1)$ corresponds a unique $y \in \mathbb{R}^n$ such that $Ax = x.y$ for every $x \in \mathbb{R}^n$. Prove that $\|A\| = |y|$
10. Show that Dirichlet's function is not Riemann integrable.
11. If $A \in L(R^n) with\ Ax \neq 0, for\ x \neq 0, is\ A\ onto$? Justify your answer.
12. Show that if f is integrable over E, then so is $|f|$ and $\left|\int_E f\right| \leq \int_E |f|$
13. Prove $\chi_{A\cap B} = \chi_A \chi_B$
14. Show that a function F is an indefinite integral then it is absolutely continuous.

(14 × 1 = 14 Weightage)

Part B

Answer any ***seven*** questions. Each question carries 1 weightage.

15. Prove that the set $[a, b]$ is not countable, where $a < b$.
16. Show that A bounded function f on $[a, b]$ is Riemann integrable if and only if the set of points at which f is discontinuous has measure zero.
17. Prove that if X is a complete metric space and ϕ is a contraction of $X \to X$ then there exist one and only one $x \in X$ such that $\phi(x) = x$

18. Show that dim $(\mathbb{R}^n) = n$.
19. Let Ω be the set of all invertable linear operators on $\mathbb{R}^n$ then Ω is an open subset of $L(\mathbb{R}^n)$ and the mapping $A \to A^{-1}$ is continuous on Ω.
20. State and prove Lebesgue convergence theorem.
21. Construct a non measurable set.
22. Is sum and product of two simple function is simple? Justify.
23. Show that the interval $(a, 5)$ is measurable.
24. Define bounded variation. Show that a function f is of bounded variation on [a,b] then f is the difference of two monotone real valued functions on [a,b].

(7 × 2 = 14 Weightage)

Part C

Answer any ***two*** questions. Each question carries 1 weightage.

25. Prove that let f be an increasing real valued function on the interval $[a, b]$. Then f is differentiable almost everywhere. The derivative f' is measurable, and $\int_a^b f'(x)dx \le f(b) - f(a)$.
26. Suppose the partial derivatives $D_j f_i$ exists and are continuous on E for $1 \le i \le m$ and $1 \le j \le n$. Show that f is continuously differentiable. Is the converse true ? Justify your answer.
27. State and prove Implicit function theorem.
28. Prove that let $\{u_n\}$ be a sequence of non negative measurable functions and let $f = \sum_{n=1}^{\infty} u_n$, then $\int f = \sum_{n=1}^{\infty} \int u_n$

(2 × 4 = 8 Weightage)

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