17P204 (Pages: 2) Name:……..………..……..…

Reg. No……………..……..

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, MAY 2018

(Regular/Supplementary/Improvement)

(CUCSS - PG)

CC15P MT2 C08 / CC17P MT2 C09 – TOPOLOGY I

(Mathematics)

(2015 Admission onwards)

Time: Three Hours Maximum:36 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. List all the Topologies on a set of 3 elements
2. Compare the strengths of different Topologies on R that you are familiar with ?
3. How do you get a topology from a sub base? Give a sub base for usual topology on R?
4. Show that $e : Y \to X$ is an embedding iff it is continuous and one-one and for every open set V in X ∃ an open set W of Y such that e(V)=W∩e(X)
5. What do you understand by a weak topology determined by a given set of functions
6. Give any three equivalent conditions for a function $f : X \to Y$ to be continuous at a point.
7. What do you mean by a weakly hereditary property?Give an example .
8. Prove that union of connected sets is connected if they have a common point
9. Show that in a connected space the only closed and open subsets are X and Φ
10. Prove that components of open subsets of a locally connected space are open
11. Show that in a Hausdorff space the limits of sequences are unique
12. Show that every map from a compact space into a T_2 space is closed
13. Does normality implies regularity? Validate your answer.
14. Let A be a subset of X and let f:A→R be continuous. Then prove that any two extensions of f to X agree on Ā.

(14 x 1 = 14 Weightage)

PART B

Answer any ***seven*** questions. Each question carries 2 weightage.

15. Define derived set A′ of a subset A of space X. Prove that Ā = A ∪ A′
16. Prove that metrisability is a hereditary property
17. Discuss the convergence of sequences in a co finite space
18. Show that every closed subset of a compact space is compact.

19. Define quotient map and a quotient space.Prove that every quotient space of a discrete space is discrete
20. Show that every open surjective map is a quotient map.
21. Differentiate between connectedness and local connectedness with examples
22. Show that the closure of a connected subset is connected.
23. Prove that a topological space X is T_1 if and only if every singleton set { x } is closed in X.
24. Let S be a subbase for a topological space X.Then show that X is completely regular iff for each V ∈ S and for each x∈ V there exists a continuous function f:X→ [0,1] such that f(x)=0 and f(y)=1for all y∉V.

(7 x 2 = 14 Weightage)

PART C

Answer any ***two*** questions. Each question carries 4 weightage.

25. Show that every continuous real valued function on a compact space is bounded and attains its extrema.
26. Show that a subset of R is connected iff it is an interval.
27. State and prove Tietze extension theorem for a function into the closed interval [-1,1]
28. Show that every regular Lindelof space is normal.

(2 x 4 = 8 Weightage)

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