17P261 (Pages: 2) Name……………….…….

Reg. No………………..

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, MAY 2018

(Regular/Supplementary/Improvement)

(CUCSS - PG)

CC15P ST2 C06 – ESTMATION THEORY

(Statistics)

(2015 Admission Onwards)

Time: Three Hours Maximum: 36 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. What do you mean by Fisher information?
2. Give an example to show that unbiased estimator of a parameter need not exist.
3. Is M.L.E unique? Justify
4. Define UMVUE and give an example.
5. Define ancillary statistic and give an example of it with proper justification.
6. Define an exponential family of distributions. Verify whether Poisson distribution is a member of this family.
7. What do you mean by Minimax estimator?
8. Define Pitman estimator.
9. Let $X \sim P(\lambda)$ Show that sample mean is CAN for λ.
10. What do you mean by efficiency of estimators? Explain.
11. Describe method of moments. Prove or disprove moment estimators are consistent.
12. Write a short note on Bayesian interval estimation.

(12 x 1 = 12 Weightage)

PART B

Answer any ***eight*** questions. Each question carries 2 weightage.

13. State and prove Rao - Blackwell theorem.
14. State and prove Fisher-Neymann factorization theorem.
15. State and Prove Basu's theorem. What is its application in Statistics?
16. State and prove Lehmann-Scheffe theorem.
17. Explain the terms (a) Bayes risk (b) Loss function (c) Posterior Distribution.
18. Examine the completeness of exponential family of distributions.
19. Define minimal sufficient Statistic. Explain a method of obtaining the same.
20. Let $X_1, X_2, ..., X_n$ be i.i.d. observations from a population with p.d.f $f(x, \theta) = \theta(1-\theta)^x$ $x = 0, 1, 2, ...$ and $0 < \theta <$ .Find Cramer Rao lower bound for the variance of an unbiased estimator of θ.
21. Let X_1, X_2 be a random sample of size two from a Poisson Population with parameter, λ Show that X_1+2X_2 is not sufficient for λ.

22. Obtain the maximum likelihood estimates of α and β for the density function $f(x) = \alpha\ e^{-\alpha(x-\beta)};\ \ \alpha > 0, \beta > 0, x > \beta$

23. Obtain the shortest confidence interval for the variance of a normal distribution based on n observations, with confidence coefficient $(1 - \alpha)$.

24. Let X follow binomial distribution with parameters n and p and assume a prior distribution of X to be uniform over (0, 1). Find the Bayes estimate and Bayes risk taking the loss function to be $L(\theta, t) = (\theta - t^2)/[\theta(1 - \theta)]$.

(8 x 2 = 16 Weightage)

PART C

Answer two questions. Each question carries 4 weightage.

25. Establish Cramer-Rao bound. Give an example to show that this bound need not be attained.

26. (a) Explain the following: (i) Shortest expected confidence interval (ii) Large sample confidence interval.

    (b) Derive the confidence interval for the parameter σ^2 in $N(\mu, \sigma^2)$

27. Define M.L.E. of a parameter. Prove that M.L.E's are asymptotically normal.

28. Apply method of moment estimation to estimate the parameter θ of the following distribution with pdf.

$$f(x;\theta) = \frac{1}{2\theta} e^{\frac{-|x|}{\theta}}, -\infty < x < \infty, \theta > 0.$$

Verify the obtained estimator is CAN estimator.

(2 x 4 = 8 Weightage)

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