18P201 (Pages: 2) Name…………….…………

Reg. No……………………..

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2019

(Regular/ Supplementary/Improvement)

(CUCSS - PG)

CC15P MT2 C06/CC17P MT2 C07/CC18P MT2 C07 - ALGEBRA II

(Mathematics)

(2015 Admission onwards)

Time: Three Hours Maximum: 36 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Find maximal ideal of $Z \times Z$
2. Is $\mathbb{Q}[x]/< x^2 - 5x + 6 >$ a field? why?
3. Prove that $x^2 - 3$ is irreducible over $\mathbb{Q}(\sqrt[3]{2})$
4. Prove that doubling the cube is impossible.
5. Find the primitive 8^{th} roots of unity in $GF(9)$
6. Find all conjugates of $\sqrt{2} + i$ over $\mathbb{R}$
7. Prove that regular 7-gon is not constructible.
8. Describe all extension of the identity map of $\mathbb{Q}$ to an isomorphism mapping $\mathbb{Q}(\sqrt[3]{2})$ onto a subfield of $\mathbb{Q}$
9. Find the degree of the splitting field of $x^4 - 1$ over $\mathbb{Q}$
10. Define a normal extension of a field F and give one example.
11. Find the 8^{th} Cyclotomic extension of $\mathbb{Q}$
12. Find the group of the polynomial $x^3 - 1$ over $\mathbb{Q}$
13. Show that the polynomial $x^5 - 2$ is solvable by radicals over $\mathbb{Q}$
14. Find $\varphi_{12}(x)$ in $\mathbb{Q}[x]$

(14 × 1 = 14 Weightage)

Part B

Answer any ***seven*** questions. Each question carries 2 weightage.

15. Prove that a field contains no proper nontrivial ideals.
16. An ideal $< p(x) > \neq \{0\}$ of $F[x]$ is maximal if and only if $p(x)$ is irreducible over F
17. If E is a finite extension field of a field F and K is a finite extension field of E, Prove that K is a finite extension of F and $[K{:}F] = [K{:}E][E{:}F]$

18. If F is any finite field, then for every positive integer n, there is an irreducible polynomial in $F[x]$ of degree n
19. Find a basis of $Q(\sqrt[3]{2}, i)$ over $\mathbb{Q}$
20. State and prove Primitive element theorem.
21. Prove that every field of characteristic zero is perfect.
22. Find $G(K/\mathbb{Q})$ where K is the splitting field of $x^3 - 2$ over $\mathbb{Q}$
23. Find all elements of the group $G(K/\mathbb{Q})$ where K is the splitting field of $x^4 + 1$ and prove that it is isomorphic to K_4
24. Let F be a field of characteristic zero, and let $a \in F$ .If K is the splitting field of $x^n - a$ over F, then $G(K/F)$ is a solvable group.

(7 × 2 = 14 Weightage)

Part C

Answer any ***two*** questions. Each question carries 4 weightage.

25. Let F be a field and let $f(x)$ be a non-constant polynomial in $F[x]$. Prove that there exist an extension field E of F and $\alpha \in E$ such that $f(\alpha) = 0$
26. Prove that field E, where $F \leq E \leq F$ is a splitting field over F if and only if every automorphism of $\overline{F}$ leaving F fixed maps E onto itself and thus induces an automorphism of E leaving F fixed.
27. State and Prove isomorphism extension theorem.
28. State main theorem of Galois. Using this Prove that G(K/F) is isomorphic to $\mathbb{Z}_{12}$ where $K = GF(p^{12})$

(2 × 4 = 8 Weightage)

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