18P203 (Pages: 2) Name………………………

Reg. No……………………..

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2019

(CUCSS - PG)

CC18P MT2 C08 – REAL ANALYSIS II

(Mathematics)

(2018 Admissions: Regular)

Time: Three Hours Maximum:36 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Is the set of rational numbers open or closed?
2. Prove that the set of rational numbers is measurable.
3. Prove that if a σ - algebra of subsets of R contains intervals of the form (a, ∞), then it contains all intervals.
4. Prove that if f is measurable, $f^2$is measurable.
5. Every subset of a set of real numbers with positive outer measure is measurable. Say true or false.
6. Give an example of a sequence of Riemann integrable functions whose limit function fails to be Riemann integrable.
7. Prove that any bounded measurable function f defined on a set of finite measure E is integrable over E.
8. Let f be a nonnegative measurable function defined on E. Then prove that $\int_E f = 0$ if and only if $f = 0$ a.e on E
9. Prove that any nonnegative function integrable over E is finite a.e. on E
10. State and prove Riesz theorem.
11. Let P be a partition of [a, b] that is a refinement of the partition P'. For a real valued function f on $[a, b]$, show that $V(f, P') \leq V(f, P)$
12. Show that a Lipschitz function f on a closed, bounded interval $[a, b]$ is absolutely continuous on $[a, b]$
13. Show that the space of all Lebesgue integrable functions $L^1(E)$ form a normed linear space with norm $\|\|_1$
14. State and prove Minkowski's inequality.

(14 x 1 = 14 Weightage)

PART B

Answer any ***seven*** questions. Each question carries 2 weightage.

15. Define a Borel set. Show that the collection of Borel sets is the smallest σ – algebra that contains the closed sets.
16. Prove that the union of a finite collection of measurable sets is measurable.
17. State and prove Lusin's theorem.
18. Let f and g be integrable over E. Then prove that for any α and β, $\alpha f + \beta g$ is integrable over E and $\int_E \alpha f + \beta g = \alpha \int_E f + \beta \int_E g$ also prove that if $f \leq g$ on E, then $\int_E f \leq \int_E g$
19. Prove the countable additive property for Lebesgue integration.
20. State and prove Vitali convergence theorem.
21. Let f be a bounded function on a set of finite measure E. Then prove that f is Lebesgue integrable over E if and only if f is measurable.
22. Let f be Lipschitz on R and g be absolutely continuous on $[a, b]$. Show that the composition $f \circ g$ is absolutely continuous on $[a, b]$
23. Prove that a function f is of bounded variation on the closed, bounded interval $[a, b]$ if and only if it is the difference of two increasing functions on $[a, b]$
24. Let E be a measurable set and $1 \leq p < \infty$. Then prove that every Cauchy sequence in $L^p(E)$ converges both with respect to the $L^p(E)$ norm and pointwise a.e. in E to a function in $L^p(E)$

(7 x 2 = 14 Weightage)

PART C

Answer any ***two*** questions. Each question carries 4 weightage.

25. Prove that the Cantor set C is a closed uncountable set of measure zero.
26. State and prove simple approximation theorem.
27. State and prove Monotone convergence Theorem.
28. Prove that a function f on a closed, bounded interval $[a, b]$ is absolutely continuous on $[a, b]$ if and only if it is an indefinite integral over $[a, b]$

(2 x 4 = 8 Weightage)

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