18P204 (Pages: 2) Name:……………………..

Reg. No:…………………..

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2019

(Regular/Supplementary/Improvement)

(CUCSS-PG)

CC15P MT2 C08 / CC17P MT2 C09 / CC18P MT2 C09 - TOPOLOGY I

(Mathematics)

(2015 Admission onwards)

Time: 3 Hours Maximum: 36 Weightage

Part-A

Answer ***all*** questions. Each question carries 1 weightage.

1. Prove that open balls in a metric space are open sets.
2. Compare ℝ with usual topology and ℝ with semi open interval topology.
3. Is the union of two topologies on a set is again a topology on that set? Justify. What about their intersection?
4. With necessary notations define product topology .
5. Define quotient map from a topological space in to another. Give an example.
6. Check whether [0,1] and the unit circle S^{ι} are homeomorphic . Justify.
7. Give an example of a topological space which is first countable but not second countable.
8. Define compactness of a topological space. Verify whether the set of reals with usual topology is compact.
9. Prove that the property of being a discrete space is divisible.
10. Define path in a topological space. Is the real line with usual topology path connected? Justify.
11. Give an example of a topological space that is T_1 but not T_2
12. Prove that every indiscrete topological space is regular.
13. Give an example of a space which is connected but not locally connected.
14. State Tietze characterisation of normality.

(14 × 1 = 14 Weightage)

Part-B

Answer any ***seven*** questions. Each question carries 2 weightage.

15. Prove that metrisability is a hereditary property.
16. State and prove the necessary and sufficient condition for a subfamily of a topology to become a base for that topology.

17. For any subset A of a space, prove that $\bar{A} = A \cup A'$

18. Suppose $(X, \mathcal{T})$ and $(Y, \mathcal{U})$ be topological spaces and $f: X \to Y$ be a function. Then prove that f is continuos if and only if $f^{-1}(V)$ is open in X for every open subset V in Y

19. Prove that every closed surjective map is a quotient map.

20. Prove that every continuous real valued function on a compact space is bounded and attains its extrema.

21. If a space X is locally connected , then prove that components of open subsets of X are open in X

22. Prove that all metric spaces are T_4

23. Prove that a compact subset in a hausdorff space is closed .

24. Let X be a completely regular space. Suppose F is a compact subset of X, C is a closed subset of X and $F \cap C = \emptyset$. Prove that there exists a continuous function from X in to the unit interval [0,1] which takes the value 0 at all points of F and the value 1 at all points of C

($7 \times 2 = 14$ Weightage)

Part-C

Answer any ***two*** questions. Each question carries 4 weightage.

25. (a) Prove that a subset A of a space X is dense in X if and only if for every nonempty open subset B of X, $A \cap B \neq \emptyset$

    (b) For a subset A of a space X, prove that

    $\bar{A} = \{ y \in X : \text{every neighbourhood of y meets A non} - \text{vacuously} \}$.

26. Prove that every closed and bounded interval is compact.

27. (a) Prove that a subset of $\mathbb{R}$ is connected iff it is an interval.

    (b) Prove that the topological product of two connected spaces is connected.

28. State and prove Urysohn characterisation of normality.

($2 \times 4 = 8$ Weightage)

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