18P263 (Pages: 2) Name:……………………..

Reg. No:…………………..

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2019

(Regular/Supplementary/Improvement)

(CUCSS-PG)

CC15P ST2 C06 - ESTIMATION THEORY

(Statistics)

(2015 Admission onwards)

Time: 3 Hours Maximum: 36 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Distinguish between parameter and statistic with examples.
2. If $f(x) = \theta x^{\theta-1}, 0 < x < 1, \theta > 0$. Find the moment estimator of θ
3. Show that T=$\sum_{i=1}^{n} X_i$ is a sufficient statistic for λ where X_i's are independent and identically Poisson distributed with parameter λ
4. If $X_1, X_2, \dots, X_n$ be a random sample of 'n' observations from $N(0, \theta)$. Find Fisher information $I_X(\theta)$
5. Define best linear unbiased estimator with an example.
6. Define CAN estimator. Give an example.
7. Let $X_1, X_2, \dots, X_n$ be a random sample of size 'n' from a population having probability density function$f(x) = e^{-(x-\theta)}, x > \theta, \theta > 0$. Find sufficient statistic for θ
8. Give an example which is the member of one parameter Cramer family of distributions.
9. If X is random variable with probability density function given by

$$f(x) = \begin{cases} \dfrac{1}{b-a}, & a < x < b \\ 0, & otherwise \end{cases}$$

with 'a' known. Find an unbiased estimator of 'b'

10. Explain Fisher information.
11. Define shortest length confidence interval.
12. Define Ancillary statistic with an example.

(12 x 1 = 12 Weightage)

PART B

Answer any ***eight*** questions. Each question carries 2 weightage.

13. Let $X_1,\ X_2,\ \dots,\ X_n$ be a random sample from a population having probability density function

$$f(x) = \begin{cases} \dfrac{2}{\theta^2}(\theta - x), & 0 \le x \le \theta \\ 0, & otherwise. \end{cases}$$

Find unbiased estimators of θ and θ^2

14. Let $X_1, X_2, \dots, X_n$ be a random sample of size 'n' from a population having PDF $f(x,\theta) = \frac{1}{\theta}, 0 < x < \theta,\ \theta > 0$. Find complete sufficient statistic for θ
15. State and Prove Fisher- Neyman Factorization theorem.
16. Let $X_i, i = 1,2, \dots, n$ be a random sample of size drawn from a Poisson distribution with parameter λ. Find $100(1-\alpha)\%$ Bayesian confidence interval for λ with the assumption of prior distribution of λ is $G(\alpha, \beta)$
17. State and prove Basu's theorem.
18. Explain the procedure of obtaining UMVUE in the presence of complete sufficient statistic.
19. State and prove Cramer-Rao inequality.
20. Distinguish between Bayesian and Fiducial interval.
21. Explain the method of construction of confidence interval using maximum likelihood estimator. Illustrate with an example.
22. Show that under certain regularity conditions , MLE is consistent.
23. Let $X_1, X_2, \dots, X_n$be a random sample of size drawn from a Poisson distribution with parameter $\boldsymbol{\lambda}$.Check the consistency and unbiasedness of the estimator

$$T = \frac{2}{n(n+1)} \sum_{i=1}^{n} X_i \text{ of } \lambda$$

24. Show that Fisher information in a statistic is always less than or at most equal to that in the original sample.

(8 x 2 = 16 Weightage)

PART C

Answer any ***two*** questions. Each question carries 4 weightage.

25. Let $X \sim N(\theta, 1)$ and prior PDF of θ be $N(0,1)$. Find the Baye's estimator of θ under squared error loss function.
26. State and Prove Lehman-Scheffe Theorem.
27. State and prove Cramer-Huzurbazar theorem.
28. Find ML estimators of the parameters of a bivariate normal distribution.

(2 x 4 = 8 Weightage)

****