

18P265

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Name.....

Reg. No.....

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2019

(Regular/Improvement/Supplementary)

(CUCSS - PG)

CC15P ST2 C08 - PROBABILITY THEORY

(Statistics)

(2015 Admission onwards)

Time: Three Hours

Maximum: 36 Weightage

Part B

Answer *all* questions. Each question carries 1 weightage.

1. Define probability space.
2. When do you say that two random variables X and Y are independent?
3. What are the properties of a distribution function?
4. Establish the relationship between convergence in probability and convergence in r th mean.
5. Does convergence in probability imply convergence in distribution and vice versa? Justify your answer.
6. State weak law of large numbers for iid random variables.
7. Define characteristic function.
8. State continuity theorem.
9. What is Radon-Nikodym derivative? Give its application.
10. State central limit theorem for iid random variables with finite mean and variance.
11. State Lindberg condition of Central limit theorem.
12. Define Martingale, sub Martingale and super Martingale.

(12 x 1 = 12 Weightage)

Part B

Answer any *eight* questions. Each question carries 2 weightage.

13. Define Axiomatic approaches of probability. Give any three consequences.
14. Check whether WLLN hold for the independent sequence, $\{X_n\}$, $P[X_n = \pm\sqrt{k}] = 1/2$
15. State and prove Kolmogorov's three series criterion.
16. State and prove Markov inequality.
17. Define tail events. State Kolmogorov's Zero-one law.
18. Give an example showing convergence in a.s. implies convergence in probability.
19. State and prove important properties of characteristic functions.

20. State and prove Helly-Bray theorem.
21. State and prove Linberg-Levy central limit theorem
22. How can we check the independence of two random variables using characteristic functions? Establish the result.
23. Prove Doob decomposition theorem for Martingales.
24. State and prove Martingale convergence theorem.

(8 x 2 = 16 Weightage)

Part C

Answer any *one* question. Each question carries 4 weightage.

25. (a) Show that convergence is almost surely implies convergence in probability
(b) Prove Borel-Cantelli lemma for independent sequence.
26. State and prove Strong Law of large numbers.
27. State and prove inversion theorem.
28. State and prove Radon-Nikodym theorem

(2 x 4 = 8 Weightage)
