D 91597 (Pages : 3) Name.....................................

Reg. No..................................

THIRD SEMESTER M.Sc. DEGREE EXAMINATION DECEMBER 2015

(CUCSS)

Mathematics

MT 3C 14—LINEAR PROGRAMMING AND ITS APPLICATIONS

Time : Three Hours Maximum : 36 Weightage

Part A (Short Answer Type)

Answer **all** *the questions.*
Each question carries a weightage of 1.

1. Define convex set. Give an example for a convex set.
2. Prove that intersection of two convex sets is a convex set.
3. Prove that a Hyperplane is a convex set.
4. Is the function $f(x) = x^2, x \in R$, a convex function. Justify your answer.
5. Distinguish between local and global extrema.
6. Define Lagrangian function and Lagrange multipliers.
7. Write the dual of the problem :

   Minimize $z = x_1 + 3x_2$ subject to $x_1 + x_2 \geq 3, -x_1 + x_2 \leq 2, x_1 - 2x_2 \leq 2, x_1 \geq 0, x_2 \geq 0$.
8. What is meant by loops in a transportation array ?
9. What is meant by unbalanced transportation problem ?
10. Describe the 0—1 variable problems in integer programming.
11. Define artificial variables. Describe the uses of artificial variables in solving linear programming problems.
12. Describe the concept of primal and dual problems in optimization theory.
13. Describe matrix games.
14. Describe the notion of dominance in game theory.

(14 × 1 = 14 weightage)

Turn over

Part B (Paragraph Type)

Answer any **seven** *questions.*
Each question carries a weightage of 2.

15. If S_F denote the set of feasible solutions of a general linear programming problem, then prove tha a vertex of SF is a basic feasible solution.

16. Use the method of Lagrange multipliers to find the maxima and minima o $x_2^2 - (x_1 + 1)^2$ subject to $x_1^2 + x_2^2 \le 1$.

17. Find the relative maxima and minima and saddle points if any of : $f(x) = x_1^3 + x_2^3 - 3x_1 - 12x_2 + 25$

18. Define the dual of a linear programming problem. Prove that if the primal problem is feasible then it has an unbounded optimum if and only if the dual has no feasible solution, and vice versa

19. Find the point in the plane $x_1 + 2x_2 + 3x_3 = 1$ in E_3 which is nearest to the point (–1, 0, 1).

20. Discuss degeneracy in transportation problems.

21. Prove that the transportation problem has a triangular basis.

22. Describe the rectangular game as a Linear programming problem.

23. Write the general form of an integer linear programming problem.

24. Explain the terms mixed strategy, pure strategy and optimal strategies with reference to an matrix game.

(7 × 2 = 14 weightage

Part C (Essay Type)

Answer any **two** *questions.*
Each question carries a weightage of 4.

25. Use simplex method to solve the problem :

Maximize $f(X) = 5x_1 + 3x_2 + x_3$ subject to the constraints

$2x_1 + x_2 + x_3 = 3, - x_1 + 2x_3 = 4, x_1 \ge 0, x_2 \ge 0, x_3 \ge 0.$

26. Solve the transportation problem for minimum cost starting with the degenerate solutio $x_{12} = 30, x_{21} = 40, x_{32} = 20, x_{43} = 60.$

	D_1	D_2	D_3	
O_1	4	5	2	30
O_2	4	1	3	40
O_3	3	6	2	20
O_4	2	3	7	60
	40	50	60	

27. Solve the following integer linear programming problem :

$$\text{Maximize } \phi(X) = 3x_1 + 4x_2 \text{ ; subject to } 2x_1 + 4x_2 \le 13,$$

$$-2x_1 + x_2 \le 2,\ 2x_1 + 2x_2 \ge 1,\ 6x_1 - 4x_2 \le 15,\ x_1, x_2 \ge 0,\ x_1 \text{ and } x_2 \text{ are integers.}$$

28. Solve the game where the pay-off matrix is $\begin{pmatrix} 2 & 7 \\ 3 & 5 \\ 11 & 2 \end{pmatrix}$.

(2 × 4 = 8 weightage)