D 91594 (Pages : 2) Name............................... 15

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THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2015

(CUCSS)

Mathematics

MT 3C 11—COMPLEX ANALYSIS

Time : Three Hours Maximum : 36 Weightage

Part A

*Answer **all** questions.*
Each question carries 1 weightage.

1. Let $T_1 z = \frac{z+2}{z+3}$; $T_2 z = \frac{z}{z+1}$. Compute $(T_1 \circ T_2)(z)$.
2. Show that if a linear transformation has ∞ for its only fixed point, then it is a translation.
3. Show that a linear transformation preserves cross ratios.
4. Find the usage of the rectangular hyperbola $\{z = x + iy : xy = 1\}$ under the map $f(z) = z^2$.
5. State Cauchy's Theorem in a disk.
6. Compute $\int_r \frac{e^z}{z-1}\, dz$, where $r(t) = 1 + e^{it}$, $0 \le t \le 2\pi$.
7. State Weierstrass' Theorem on essential singularity.
8. Show that if f is analytic in a region G and if $f \ne 0$, then the zero of f are isolated.
9. Find the poles and residues of the function $\frac{1}{\left(z^2 - 1\right)^2}$.
10. State the Maximum Principle for harmonic functions.
11. How many roots does the equation $z^7 - 2z^4 + 6z^2 - z + 1 = 0$ have in the disk $\{z : |z| < 1\}$?
12. Obtain the power series expansion of $\frac{1}{z+3}$ about $z = 1$ in the disk $\{z : |z-1| < 4\}$.
13. Show that an elliptic function without poles is a constant.
14. Show that a non-constant elliptic function has equally many poles as it has zeros.

(14 × 1 = 14 weightage)

Turn over

Part B

Answer any **seven** *questions.*
Each question carries 2 marks.

15. Show that the cross ratio (z_1, z_2, z_3, z_4) is real if and only if the four points lie on a circle or a straight line.
16. Describe the mapping properties of $w = e^z$.
17. Prove that a bounded entire function reduces to a constant.
18. Let r be a closed rectifiable curve. Prove that $n(r, z)$ is a constant in each of the regions determined by r.
19. State and prove Schwarz's lemma.
20. Suppose f is analytic in a region Ω and statistics the inequality $|f(z) - 2| < 2$ in Ω. Show that

    $$\int_r \frac{f'(z)}{f(z)}\, dz = 0 \text{ for every closed curve } r \text{ in } \Omega.$$

21. State and prove Hurwitz theorem.
22. Obtain the Laurent series expansion of $\frac{1}{z(z-1)(z-2)}$ in the regions :

    (i) $0 < |z| < 1$; (ii) $1 < |z| < 2$; and (iii) $|z| > 2$.

23. State and prove Rouche's Theorem.
24. Derive the Legendre relation :

    $\eta_1 w_1 - \eta_2 w_2 = 2\pi i.$

(7 × 2 = 14 weightag

Part C

Answer any **two** *questions.*
Each question carries 4 weightage.

25. State and prove Cauchy's theorem for a rectangle.
26. State the residue theorem. Explain how it can be applied to calculate real integrals. Illustrate wi an example.
27. Derive the Poisson integral formula for harmonic functions.
28. Derive the formula for the Weierstrass elliptic function P (z) in the form :

    $$P(z) = \frac{1}{z^2} + \sum_{w \neq 0}\left(\frac{1}{(z-w)^2} - \frac{1}{w^2}\right).$$

(2 × 4 = 8 weighta