16P301 (Pages : 2) Name……………………

Reg. No…………………

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, OCTOBER 2017

(Regular/Supplementary/Improvement)

(CUCSS - PG)

CC15PMT3C11 - COMPLEX ANALYSIS

(Mathematics)

(2015 Admission Onwards)

Time: Three Hours Max : 36 weightage

PART- A

Answer **ALL** questions Each question carries 1 weightage

1. Define conformal mapping with an example.
2. Show that Linear Transformation preserves cross ratio.
3. Find the cross ratio of $(1,\ i,\ \infty,\ 0)$.
4. Find the image of hyperbola $\{ z = x + i\,y : x^2 - y^2 = 1\}$ under the map $f(z) = z^2$.
5. State Cauchy's integral formula.
6. If f is analytic in a region G and if $f \neq 0$ then zero's of f are isolated.
7. What is the nature of the singularity of e^z at $z = \infty$.
8. Compute $\int_{|z|=1} \frac{1}{z^2+1}\, dz$.
9. Find the residues of the function $f(z) = \tan z$ at $z = \frac{\pi}{2}$.
10. How many roots does the equation $z^4 - 6z + 3 = 0$ have in the annulus $1 < |z| < 2$.
11. State maximum principle for harmonic functions.
12. Expand $\frac{6}{(z-2)(z-3)}$ as a Laurent series in the region $2 < |z| < 3$.
13. Prove that a non constant elliptic function has equally many poles as its zeros.
14. Find the harmonic conjugate of the function $e^x \cos y$.

(14×1=14 weightage)

PART- B

Answer any **SEVEN** questions Each question carries 2 weightage

15. Find the linear transformation which carries $0,\ i,\ -i$ to $1, -1,\ 0$.

16. Prove that the cross ratio (z_1, z_2, z_3, z_4) is real if and only if the four points lie on a circle or on a straight line.

17. Let γ be a closed rectifiable curve. For any point a not on γ define $n(\gamma\ , a)$. Show that $n(\gamma, a)$ is always an integer.

18. State and prove Morera's theorem.

19. State and prove Schwarz lemma.

20. State and Prove Cauchy's residue theorem.

21. If f is analytic in a region G and $\boldsymbol{f}'\,(z) \neq 0$ for any z in G, prove that log $|f(z)|$ is harmonic in G.

22. Evaluate $\int_0^\infty \frac{dx}{(1+x^2)^2}$.

23. Derive the Legendre relation $n_1\omega_1 - n_2\omega_2 = 2\pi i$.

24. Prove that $\begin{vmatrix} \wp(z) & \wp'(z) & 1 \\ \wp(u) & \wp'(u) & 1 \\ \wp(u+z) & -\wp'(u+z) & 1 \end{vmatrix} = 0.$

(7 × 2=14 weightage)

PART- C

Answer any **TWO** questions Each carries 4 weightage

25. State and prove Cauchy's theorem for a disc.
26. Derive Poisson integral formula for harmonic function.
27. Discuss the evaluation of integrals of the type $\int_{-\infty}^{\infty} R(x)e^{ix}dx$ using the theory of residues.
28. State and prove Taylor's theorem.

(2 × 4=8 weightage)

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