16P303 (Pages:2) Name……………………

Reg. No………………..

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, OCTOBER 2017

(Regular/Supplementary/Improvement)

(CUCSS - PG)

CC15P MT3 C13 - TOPOLOGY II

(Mathematics)

(2015 Admission Onwards)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** questions.

Each question carries 1 weightage.

1. Define a box.
2. Let (X, τ) be the topological product of an indexed family of topological spaces $\{(X_i, \tau_i): i\epsilon I\}$ and let Y be any topological space. Then show that a function $f: X \rightarrow Y$ is continuous if and only if for each $i \in I$, the composition $\pi_i o f: Y \rightarrow X_i$ is continuous.
3. Define countably productive property. Give an example.
4. Show that a topological product is T_o if and only if each coordinate space has the corresponding property.
5. Define evaluation function of the indexed family $\{f_i: i \in I\}$ of functions.
6. Give an example of a metric space which is not second countable.
7. Define path homotopy.
8. Let X be a space. Define first homotopy group of X.
9. Define covering map and give an example of it.
10. Define a strong deformation retract of a space X.
11. Show that a first countable countably compact space is sequentially compact.
12. Give an example of a countably compact space.
13. Define Stone-Cech compactification of a topological space X.
14. Prove that $\mathbb{R}$ with usual topology is of second category.

(14 x1=14 weightage)

Part B

Answer **any 7** questions.
Each question carries 2 weightage

15. Show that a subset of X is a box if and only if it is the intersection of a family of walls.
16. Prove that projection functions are open.
17. Prove that a topological product is regular if and only if each coordinate space has the corresponding property.
18. Let (X, d) be a metric space and let μ be any positive real number. Show that there exists a metric e on X such that $e(x, y) \leq \mu$ for all $x, y \epsilon X$ and e induces the same topology on X as d does.
19. State and prove embedding lemma.
20. Let X be path connected and x_0 and x_1 be two points of X. Prove that $\pi_1(X, x_0)$ is isomorphic to $\pi_1(X, x_1)$.
21. Prove that the map p:$\mathbb{R}$→S^1 given by the equation $\mathrm{p(x)} = (\cos 2\pi\mathrm{x}, \sin 2\pi\mathrm{x})$ is a covering map.
22. Prove that every countably compact metric space is second countable.
23. Prove that if Y is a compact Hausdorff space and y_o is any point of Y then Y is homeomorphic to the Alexandroff compactification of the space $Y - \{y_o\}$.
24. Let $(X; d)$ be a complete metric space. Prove that a subset of first category in X cannot have any interior points.

(7 x2=14 weightage)

Part C

Answer any **two** questions.
Each question carries 4 weightage

25. Prove that product of topological spaces is connected if and only if each coordinate space is connected.
26. State and prove Urysohn's metrisation theorem.
27. Prove that any continuous function from a Tychonoff space into a compact Hausdorff space can be extended continuously over the Stone-Cech compactification of the domain.
28. Prove that a metric space is compact if and only if it is complete and totally bounded.

(2 x4=8 weightage)

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