

17P306

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Name:

Reg. No.....

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(CUCSS - PG)

CC17P MT3 C13 - COMPLEX ANALYSIS

(Mathematics)

(2017 Admission)

Time: Three Hours

Maximum: 36 Weightage

Part A

Answer *all* questions. Each question carries 1 weightage.

1. Define conformal mapping with example.
2. If γ lies inside a circle then prove that $n(\gamma, a) = 0$ for all points a outside the same circle.
3. Prove $\int_{-\gamma} f(z) dz = - \int_{\gamma} f(z) dz$ where γ is a piecewise differentiable arc.
4. Evaluate $\int_{\gamma} y dz$ where γ is the directed line segment from 0 to $1 + i$.
5. State Schwarz Lemma.
6. Define isolated and non isolated singularities with example.
7. Define residue of $f(z)$ at an isolated singularity with example.
8. Show that $\int_{\gamma} * du = 0$ for every cycle $\gamma \sim 0 \pmod{\Omega}$.
9. Define Simply connected region. Give an example of a simply connected region.
10. Find the residue of the function $f(z) = \frac{z^2-2}{(z-2)^2}$ at $z = 2$.
11. Show that a non constant analytic function maps open sets into open sets.
12. State the maximum principle of harmonic functions.
13. How many roots does the equation $z^7 - 2z^5 + z^3 - z + 1 = 0$ have in the disc $|z| < 1$.
14. Find the Taylor series expansion of the function $\frac{1}{z-2}$ at $z=1$.

(14 × 1 = 14 Weightage)

Part B

Answer any *seven* questions. Each question carries 2 weightage.

15. A linear transformation carries \mathbb{R} into \mathbb{R} if and only if it can be written with real coefficients.
16. Prove that an analytic function comes arbitrarily close to any complex value in every neighbourhood of an essential singularity.
17. Evaluate $\int_{|z|=\rho} \frac{|dz|}{|z-a|^2}$ where $|a| \neq \rho$.

18. State and prove Liouville's Theorem.
19. Let f be a continuous complex valued function defined on the closed interval $[a, b]$. Prove that $\left| \int_a^b f(t) dt \right| \leq \int_a^b |f(t)| dt$.
20. If a piecewise differentiable closed curve γ does not pass through the point a then the value of the integral $\int_{\gamma} \frac{dz}{z-a}$ is an integral multiple of $2\pi i$.
21. Prove that a linear transformation carries circles into circles.
22. Suppose f has an isolated singularity at $z = a$. If $\lim_{z \rightarrow a} (z - a)f(z) = 0$. Show that $z = a$ is a removable singularity.
23. Prove that the Laurent development is unique.
24. State and prove the generalized form of argument principle.

(7 × 2 = 14 Weightage)

Part C

Answer any *two* questions. Each question carries 4 weightage.

25. Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5+4 \cos \theta} d\theta$.
26. State and prove the general statement of Cauchy's Theorem.
27. The cross ratio (z_1, z_2, z_3, z_4) is real if and only if the four points lie on a circle or on a straight line.
28. Derive the Poisson integral formula for harmonic functions.

(2 × 4 = 8 Weightage)
