17P305 (Pages: 2) Name……………………..

Reg. No…………………..

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(CUCSS-PG)

(Mathematics)

CC17P MT3 C12 - MULTIVARIABLE CALCULUS AND GEOMETRY

(2017 Admission)

Time: Three Hours Maximum : 36 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Define Linear Operator on Space X. When can we say that a given Operator is Invertible.
2. Prove that every basis of a finite dimensional Vector Space has the same number of vectors.
3. X is a set consisting of $\mathbf{0}$ alone. X is a finite dimensional vector space with $\dim X = 1$. State True or False. Justify.
4. For $A \in L(\mathcal{R}^n)$ define the operator norm.
5. Prove that $|Ax| \leq \|A\| \ |x|$ for all $x \in \mathcal{R}^n$
6. State Inverse function Theorem.
7. Find the tangent vectors of the unit circle.
8. Define the signed curvature of a cure by giving suitable example.
9. Compute the curvature of $\Upsilon(t) = (Cos^3 t, Sin^3 t)$
10. Define diffeomorphism between 2 smooth surfaces.
11. Calculate the first fundamental form of $\sigma(u, v) = (u - v, u + v, u^2 + v^2)$
12. Define the second fundamental form of the surface.
13. Define Weingarten map at a point P on a surface S
14. Define Gaussian and Mean Curvatures.

$(\mathbf{14 \times 1 = 14\ Weightage})$

PART B

Answer any ***seven*** questions. Each question carries 2 weightage.

15. If A is an invertible linear operator on $\mathcal{R}^n$, the prove that A^{-1} is linear and Invertible.
16. If S is a non empty subset of a vector space V, prove that $Span\ S$ is vector space.
17. Prove that a linear operator A on a finite dimensional vector space X is one to one iff the range of A is full of X

18. If X is a complete metric space, anf if φ is a contraction of X into X, then there exists one and only one $x \in X$ such that $\varphi(x) = x$
19. Prove that the curvature of a circular helix is a Constant.
20. Prove that a parameterized curve has a unit speed reparametrization iff it is regular.
21. Prove that the total signed curvature of a closed plane curve is an integer multiple of 2π
22. Show that if a curve Υ is T_1 periodic and T_2 periodic, then it is $K_1T_1 + K_2T_2$ periodic for any integer K_1 and K_2
23. Prove that the first fundamental form is an example of an Inner product.
24. Prove that the Weingarten map is Self-adjoint.

($7 \times 2 = 14$ Weightage)

PART C

Answer any ***two*** questions. Each question carries 4 weightage.

25. Let $f: E \to \mathcal{R}^m$, where E is an open subset of $\mathcal{R}^n$
    i. When do we say that f is differentiable at $\boldsymbol{x} \in E$?
    ii. Establish the uniqueness of the derivative.
    iii. If f is differentiable at $\boldsymbol{x} \in E$, for $\boldsymbol{h} \in \mathcal{R}^n$ prove that
    $\boldsymbol{f}'(\boldsymbol{x})\boldsymbol{h} = \sum_{i=1}^{m}\{\sum_{j=1}^{n}(D_jf_i)(\boldsymbol{x})h_j\}\boldsymbol{u}_i$ Where $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \dots, \boldsymbol{u}_m\}$ is the standard basis of $\mathcal{R}^m$
26. If $\boldsymbol{f}: E \to \mathcal{R}^n$, where E is an open subset of $\mathcal{R}^n$ is a continuously differentiable mapping and if $\boldsymbol{f}'(\boldsymbol{x})$ is invertible for every$\boldsymbol{x} \in E$, then prove that $\boldsymbol{f}(W)$ is an open subset of $\mathcal{R}^n$ for every open set $W \subset E$
27. Find the atlas for the following.
    i. $S = \{(x, y, z) \in \mathcal{R}^3 : x^2 + y^2 = 1\}$
    ii. $S = \{(x, y, z) \in \mathcal{R}^3 : x^2 + y^2 + z^2 = 1\}$
28. State and Prove Euler's theorem on Principal Curvature.

($2 \times 4 = 8$ Weightage)

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