17P308 (Pages: 2) Name……………………..

Reg. No…………………..

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(CUCSS - PG)

CC17P MT3 C15 - PDE AND INTEGRAL EQUATIONS

(Mathematics)

(2017 Admission)

Time: Three Hours Maximum: 36 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Determine a partial differential equation of first order satisfied by the surface $F(u,v) = 0$, where $u = u(x,y,z)$ and $v = v(x,y,z)$ are known functions of x, y and z and F is an arbitrary function of u and v.
2. Eliminate the parameter a and b from the equation $z = ax + by$ and find the corresponding partial differential equation.
3. Solve $xp + yq = z$.
4. If $\bar{X}.curl\bar{X} = 0$, where $\bar{X} = (P, Q, R)$ and μ is an arbitrary differentiable function of $x, y, z,$ then prove that $\mu\bar{X}.curl\mu\bar{X} = 0$
5. Find the complete integral of $p + q - pq = 0$
6. What is characteristic strip?
7. Write the classification of the equation $u_{xx} + xu_{yy} = 0$ in the region $x < 0$
8. What is Riemann function?
9. State the Neumann problem for a circle.
10. Show that the solution of the Dirichlet problem, if it exist is unique.
11. Define Volterra equation of second kind and give an example.
12. Show that if y(x) satisfy the differential equation $\frac{d^2y}{dx^2} + xy = 1$ and the conditions $y(0) = y'(0) = 0$, then y satisfies the Volterra equation $y(x) = \int_0^x \xi(\xi - x)y(\xi)d\xi + \frac{x^2}{2}$
13. Determine the resolvant kernel associated with $K(x,\xi) = x + \xi$ in (0,1) in the form of the power series obtaining first 2 terms.
14. Consider $y'' + xy = 1$ with $y(0) = y(l) = 0$. Find the Green's function.

(14 x 1 = 14 Weightage)

Part B

Answer any ***seven*** questions. Each question carries 2 weightage.

15. Find the general integral of $x^2p + y^2q = (x + y)z$

16. Verify that the Pfaffian differential equation
$(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$ is integrable and find the corresponding integrals.
17. Solve $z^2 - pqxy = 0$
18. Solve by Jacobi's method $p^2x + q^2y = z$
19. Reduce the equation $u_{xx} - x^2u_{yy} = 0$ to its canonical form.
20. Suppose that $u(x, y)$ is harmonic in a bounded domain $\mathfrak{D}$ and continuous in $\bar{\mathfrak{D}} = \mathfrak{D} \cup B$, then u attains its maximum on the boundary B of $\mathfrak{D}$
21. Show that the solution of the Neumann problem is unique upto the addition of a constant.
22. If $y''(x) = F(x)$ and y satisfies the conditions y (0) = 0 and y(1) = 0. Show that $y(x) = \int_0^1 K(x,\xi)F(\xi)d\xi$ where $K(x,\xi) = \begin{cases} \xi(x-1), & \xi < x \\ x(\xi - 1), & \xi > x \end{cases}$. Also verify that this expression satisfies the prescribed differential equation and end conditions.
23. Write a note on Neumann series.
24. Determine the coefficient of λ in the expansion of resolvant kernel associated with $K(x, \xi) = e^{|x-\xi|}$ in (0,a)

(7 x 2 = 14 Weightage)

Part C

Answer any ***two*** questions. Each question carries 4 weightage.

25. Show that $(x - a)^2 + (y - b)^2 + z^2 = 1$ is a complete integral of $z^2(1 + p^2 + q^2)1$
By taking $b = 2a$ show that the envelope of the subfamily is $(y - 2x)^2 + 5z^2 = 5$ which is a particular solution. Show further that $z = \pm 1$ are the singular integrals.
26. Find the characteristic strips of the equation $xp + yq - pq = 0$ and obtain the equation of the integral surface through the curve $C: z = \frac{x}{2}, y = 0$
27. State and prove the heat conduction problem in an infinite rod.
28. Consider the equation $y(x) = F(x) + \lambda \int_0^{2\pi} \cos(x + \xi)\, y(\xi)d\xi$
    a. Determine the characteristic values of λ and corresponding functions.
    b. Obtain the solution when F(x) = sinx considering all possible cases.

(2 x 4 = 8 Weightage)

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