17P371 (Pages: 2) Name: ……………………..

Reg. No……………………….

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(Regular/Supplementary/Improvement)

(CUCSS - PG)

CC15P ST3 C11 - STOCHASTIC PROCESSES

(Statistics)

(2015 Admission onwards)

Time : Three Hours Maximum : 36 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Define stationary stochastic process.
2. Write an example for continuous time continuous state stochastic process
3. State properties of transition probability matrix of a Markov chain.
4. State and prove the memory less property of exponential distribution
5. Define a Poisson process
6. Describe pure Birth process.
7. Show that the renewal function is $m(t) = \sum_{n=1}^{\infty} F_n(t), \forall t$, where $F_n(t) = \mathrm{P}(S_n \leq t), n \geq 1, \forall\, t$.
8. Define renewal reward process.
9. Distinguish between M/M/1 Queing model and M/G/1 Queing Model.
10. Define Brownian motion process.
11. What are the elementary properties of a Weiner process?
12. What is offspring distribution?

(12 x 1 = 12 Weightage)

PART B

Answer any ***eight*** questions. Each question carries 2 weightage.

13. Prove that Markov chain is completely determined by the one-step transition probability matrix and the initial distribution.
14. Show that state i is recurrent if $\sum_{n=1}^{\infty} p_{ii}^{(n)} = \infty$ and is transient if $\sum_{n=1}^{\infty} p_{ii}^{(n)} < \infty$.
15. Let $\{X_n, n = 1, 2, \ldots\}$ be a four step Markov chain with one step transition probability matrix $\begin{bmatrix} .3 & .7 & 0 \\ 0 & .5 & .5 \\ .7 & .3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Find the periodicities of the states.

16. Prove that the interval between two successive occurrences of a Poisson process follow exponential distribution.
17. Find the Chapman – Kolmogorov equation for discrete time Markov chain.
18. If $\{N(t)\}$ is a renewal process, Show that the number of renewals by time $t \geq n$ if and only if the n^{th} renewal occurs on or before time t.
19. State and prove the first entrance theorem.
20. Explain the semi-Markov process.
21. Derive the steady state probabilities of M/M/s model.
22. Show that in an irreducible Markov chain, all states are of same type.
23. Derive the distribution of first hitting time of a Brownian motion process.
24. When do you say that a state is in absorbing stage? Give example.

(8 x 2 = 16 Weightage)

PART C

Answer any ***two*** questions. Each question carries 4 weightage.

25. State recurrence, transience and class-properties with example. Derive the relationship between Ergodicity and stationary distribution.
26. Define branching process. Find the mean and variance of the G.W. branching process. State and prove elementary renewal theorem.
27. Let S_n be the waiting time for the occurrence of n^{th} renewal and $m(t)$ be the renewal function of renewal process. Find $E\{S_{N(t)+1}\}$. Show that $E\{S_{N(t)+1}\} = E(X_1)\{1 + m(t)\}$.
28. Find the balance equations in M/M/1 model and Expected number in the system.

(2 x 4 = 8 Weightage)

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