17P372 (Pages: 2) Name: ……….……………..

Reg. No…………………….

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(Regular/Supplementary/Improvement)

(CUCSS - PG)

CC15P ST3 C12 -TESTING OF STATISTICAL HYPOTHESES

(Statistics)

(2015 Admission onwards)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Define non randomized Test.
2. State Generalized Neyman-Pearson lemma.
3. What is level of significance?
4. Show that for Neyman-Pearson tests, power is greater than the size of test.
5. What are UMP unbiased tests?
6. Discuss the principle of invariance in testing of hypothese.
7. What are Bayesian tests?
8. Briefly describe χ^2-test for testing the independent of attributes.
9. Define median test.
10. State Karlin-Rubin theorem.
11. What are the advantages of SPRT over fixed sample test?
12. Briefly describe important properties of SPRT.

(12 x 1 = 12 Weightage)

Part B

Answer any ***eight*** questions. Each question carries 2 weightage.

13. Let $X \sim U(0, \theta)$ based on n observations on X, derive the most powerful test for testing $H_0: \theta = \theta_0$ v/s $H_0: \theta = \theta_1$ $(\theta_0 < \theta_1)$
14. What is MLR property? Verify whether the Laplace distribution with pdf f $(x) = \frac{1}{2} \exp(-|x - \theta|), -\infty < x < \infty, \theta \epsilon \mathbb{R}$, possess MLR property.
15. What are UMP tests? Give an example where (i) UMP test does not exist, (ii) UMP test exist.
16. Suppose that $X_1, ..., X_n$ are iid random variables having the Poisson(λ) distribution where $\lambda \in \Re^+$ is the unknown parameter. With preassigned $\alpha \in (0, 1)$, derive the randomized UMP level α test for $H_0 : \lambda = \lambda_0$ versus $H1 : \lambda < \lambda_0$ where λ_0 is a positive number.

17. Describe Likelihood ratio tests and discuss its properties.
18. Suppose that $X_1, ..., X_n$ are iid $N(0, \sigma^2)$ where $\sigma(> 0)$ is the unknown parameter. With preassigned $\alpha \in (0, 1)$, derive a level α LR test for the null hypothesis $H_0 : \sigma^2 = {\sigma_0}^2$ against an alternative hypothesis $H_1: \sigma^2 \neq {\sigma_0}^2$ in the implementable form.
19. What are α-similar tests? Discuss the construction of α-similar tests with Neyman structure.
20. Explain Mann-Whiteny test for two sample problem.
21. What is Kolmogrov-Smirnov test? Discuss its applications.
22. Define Kendall's tau. Describe properties of Kendall's tau.
23. Define OC and ASN function. Describe its properties.
24. Show that for a SPRT with stopping bounds A and B, A >B, and strength (α, β)

    $A \leq \frac{1-\beta}{\alpha}$ *and* $B \geq \frac{\beta}{1-\alpha}$.

(8 x 2 = 16 Weightage)

Part C

Answer any ***two*** questions. Each question carries 4 weightage.

25. (a) Distinguish between randomised tests and non-randomised tests.

    (b) Suppose that $X_1, ..., X_n$ are iid Geometric(p) where $p \in (0, 1)$ is the unknown parameter. With preassigned $\alpha \in (0, 1)$, derive the randomized UMP level α test for $H_0 : p \geq p_0$ versus $H_1 : p < p_0$ where p_0 is a number between 0 and 1.
26. Describe locally most powerful tests. Suppose $(X_1, . . ., X_n)$ is a random sample from a $N(\theta, 1)$ distribution. Show that the locally most powerful test of $H_0 : \theta = 0$ against $H_1 : \theta > 0$ is also the uniformly most powerful test.
27. (a) Describe sign test.

    (b) Explain Wilcoxon signed rank test. What are the advantages of Wilcoxon signed rank test over sign test?
28. Explain SPRT. Derive SPRT for testing H_0: $\theta = \theta_0$ vs H_1: $\theta = \theta_1$ for $N(0, \theta)$. Derive the expression for OC and ASN functions in this case.

(2 x 4 = 8 Weightage)

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