18P306 (Pages: 2) Name:..............................

Reg. No. ....

THIRD SEMESTER M.Sc. DEGREE EXAMINATION NOVEMBER 2019

(CUCSS-PG)

CC18P MT3 C03 - COMPLEX ANALYSIS

(Mathematics)

(2018 Admission Regular)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Let S the Riemann sphere. For the points $1+i$ and $2+3i$ in $\mathbb{C}$, give the corresponding points in S.
2. Let $\gamma : [0, 2\pi] \to \mathbb{C}$ be defined by $\gamma(t) = e^{it}$. Evaluate $\int_\gamma \frac{1}{z} dz$.
3. Find the Mobius transformation which maps the points $1, i, -1$ into the points $i, 0, -i$.
4. Describe the set $\{z \in \mathbb{C} : e^z = i\}$.
5. Find the fixed points of a dilation.
6. Evaluate the integral $\int_\gamma \frac{2z+1}{z^2+z+1} dz$ where γ is the circle $|Z| = 2$.‘
7. Evaluate the integral $\int_\gamma \frac{e^z - e^{-z}}{z^n} dz$ where n is a positive integer and $\gamma(t) = e^{it}$, $0 \leq t \leq 2\pi$
8. Let γ be closed rectifiable curve in an open set G. Show that if γ is homotopic to zero in G then γ is homologous to zero in G.
9. Show that if f and g are analytic functions on a region G such that $f.g(z) = f(z)g(z) = 0$ for all $z \in G$, then either $f \equiv 0$ or $g \equiv 0$.
10. Define essential singularity. Give an example.
11. Let $f(z) = \frac{1}{z(z-1)(z-2)}$. Give the Laurent expansion of $f(z)$ in ann $(0;\, 1, 2)$.
12. Prove that if $f : G \to \mathbb{C}$ is analytic and one-one, then $f'(z) \neq 0$ for any $z \in G$.
13. Let $D = \{z : |z| < 1\}$ and f be analytic on D with $|f(z)| \leq 1$ for all z in D and $f(0) = 0$. Show that $|f(z)| \leq |z|$ for all z in D.
14. Determine the nature of the singularity of the function $f(z) = \frac{sinz}{z}$ at $z = 0$.

($14 \times 1 = 14$ Weightage)

Part B

Answer any ***seven*** questions. Each question carries 2 weightage.

15. Let $\gamma : [a, b] \to \mathbb{C}$ be a piece wise smooth function and $f : [a, b] \to \mathbb{C}$ be continuous. Prove that $\int_a^b f d\gamma = \int_a^b f(t) d\gamma(t) dt$.
16. Discuss the mapping properties of the function $f(z) = z^2$.
17. Let G he an open connected subset of $\mathbb{C}$ and $f : G \to \mathbb{C}$ be differentiable with $f'(z) = 0$ for all $z \in G$. Show that f is a constant function.
18. If f is analytic in $B(a, R)$ and $|f(z)| \leq M$ for all z in $B(a, R)$, prove that $|f^n(a)| \leq \frac{n!M}{R^n}$.
19. Let $\gamma : [a, b] \to \mathbb{C}$ be a closed rectifiable curve and $a \notin \{\gamma\}$. Prove that $\frac{1}{2\pi i} \int_\gamma \frac{dz}{(z-a)}$ is an integer.
20. Let G be a region and let $f : G \to \mathbb{C}$ be a continuous function such that $\int f = 0$ for any triangular path T in G. Show that f is analytic.
21. Let G be a simply connected and $f : G \to \mathbb{C}$ be an analytic function such that $f(z) = 0$ for any $z \in G$. Show that there is an analytic function $g : G \to \mathbb{C}$ such that $f(z) = exp\, g(z)$.
22. Find poles of the function $f(z) = \frac{z^2}{1+z^4}$ and determine residue of $f(z)$ at one of its poles.
23. State Rouche's theorem and deduce fundamental theorem of Algebra.
24. Let G be a region and f be a non constant analytic function on G. Show that $f(U)$ is open for any open set U in G.

($7 \times 2 = 14$ Weightage)

Part C

Answer any ***two*** questions. Each question carries 4 weightage.

25. (a) Prove that the power series $\sum_{n=0}^{\infty} a_n (z-a)^n$ converges absolutely for each $z \in B(a, R)$ where $\frac{1}{R} = lim\, sup\, |a_n|^{\frac{1}{n}}$.

    (b) Find the radius of convergence of the following power series

    (i) $\sum_{n=0}^{\infty} a^{n^2} z^n$. (ii) $\sum_{n=0}^{\infty} z^n$

26. Let γ_0 and γ_1 be two closed rectifiable curves in a region G and γ_0 and γ_1 be homotopic, then show that $\int_{\gamma_0} f = \int_{\gamma_1} f$ for every function f analytic on G.
27. State and prove Laurent Series Development.
28. Evaluate the integral $\int_0^\infty \frac{sinx}{x}$.

($2 \times 4 = 8$ Weightage)

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