18P307 (Pages:2) Name……………………..

Reg. No…………………..

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(Regular/Supplementary/Improvement)

(CUCSS-PG)

(Mathematics)

CC17P MT3 C14/ CC18P MT3 C14 - FUNCTIONAL ANALYSIS I

(2017 Admission onwards)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Define a complete metric space. Give an example.
2. Show that the metric space l^{∞} is not separable.
3. Define Fourier series of a 2π-periodic function. Write down the Fourier series of the function,

$$x(t) = sin^2(t), \qquad t \in [-\pi, \pi]$$

4. State and prove Jenson inequality.
5. Give an example of a discontinuous linear functional on a normed space.
6. Let $X = R^2$ with the norm $\|\ \ \|_\infty$ and $Y = \{(x(1), x(2)) \in X : x(1) = 0\}$ and $g \in Y'$ defined by $(x(1), x(2)) = x(2)$. Find a Hahn- Banach extension of g to X
7. Show that C_{00} can not be a Banach space in any norm.
8. Define a Schauder basis and give one example.
9. Let X be a normed space and E be a subset of X such that $f(E)$ is bounded for every $f \in X'$. Show that E is bounded.
10. Let X be an inner product space and $x \in X$. Show that x = 0 if and only if $\langle x, y\rangle = 0$ for each $y \in X$
11. Let E be an orthogonal subset of an inner product space and 0 $\notin$ E. Show that E is linearly independent.
12. Let H be a Hilbert Space, $\{u_1, u_2 \dots \dots \dots \dots \dots \dots .\}$ is a countable orthonormal set in H and k_1, k_2……..are scalars such that $\sum_n |k_n^2| < \infty$. Show that $\sum_n k_n u_n$ converges in H
13. Let X be an inner product space, $\{x_1, x_2, \dots\dots\dots x_n\}$ be an orthonormal set in X and $k_1, k_2, \dots\dots\dots k_n \in K$ be such that $\|k_j\|$ = 1 for j = 1, 2, ….n. Show that

$$\|k_1x_1 + k_2x_2 + \cdots \dots \dots . + k_nx_n\| = \|x_1 + x_2 + \cdots \dots \dots + x_n\|$$

14. State and prove Polarization identity on a linear space X

(14 x 1 = 14 Weightage)

Part B

Answer any ***seven*** questions. Each question carries 2 weightage.

15. State and prove Minkowski's inequality.
16. Define the n^{th} Dirichlet kernel $D_n(t)$. Evaluate $\int_{-\pi}^{\pi} D_n(t)dt$. Also Show that $\int_{-\pi}^{\pi}|D_n(t)dt| \to 0$ as $n \to \infty$
17. Let X be a normed space. Show that every closed and bounded subset of X is compact if and only if X is finite dimensional.
18. Show that a linear map on a linear space may be continuous with respect to some norm and may be discontinuous with respect to some other norm.
19. State and prove Hahn -Banach extension Theorem.
20. Let X and Y be a normed spaces and $X \neq \{0\}$. Show that Y is a Banach space if and only if BL (X, Y) is a Banach space in the operator norm.
21. State and prove Schwarz inequality.
22. Let X be a normed space with the norm. $\| \quad \|$, which satisfies the parallelogram law. Show that $\| \quad \|$ is induced by an inner product.
23. If a Hilbert space H is seperable, show that, it has a countable orthonormal basis.
24. Let X be a normed space over K, $f \epsilon X'$ and $f \neq 0$. Let $a \epsilon X$ with $f(a) = 1$ and $r > 0$. Then $U(a,r) \cap Z(f) = \emptyset$ if and only if $\|f\| \leq \frac{1}{r}$

(7 x 2 = 14 Weightage)

Part C

Answer any ***two*** questions. Each question carries 4 weightage.

25. For $1 \leq p \leq \infty$, show that the metric space L^p(E) is complete.
26. a) If X is a finite dimensional normed space, Y any normed space and $F : X \to Y$ is linear. Show that F is continuous.

    b) State and prove Bessel's Inequality.
27. State and prove closed graph theorem.
28. State and prove uniform boundedness principle.

(2 x 4 = 8 Weightage)

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