18P305 (Pages: 2) Name……………………..

Reg. No…………………..

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(Regular/Supplementary/Improvement)

(CUCSS-PG)

CC17P MT3 C12/CC18P MT3 C12 - MULTIVARIABLE CALCULUS AND GEOMETRY

(Mathematics)

(2017 Admission onwards)

Time: Three Hours Maximum: 36 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Let A $\in L(R^n, R^m)$.Then prove that A is a uniformly continuous mapping of $R^n \; to \; R^m$
2. Let $A : R^2 \to R^2$ be defined by $A(x, y) = (3x + 2y, 4x - 7y)$. Find the derivative of A at any point (x, y)
3. Let $f: R^2 \to R^3$ be given by $f(x, y, z) = x^2 + y^2 + z^2$. Find the directional derivative of f at (1,0,1) in the direction of the vector $\left(\frac{3}{5}, 0, \frac{4}{5}\right)$
4. Give examples of contractions on $(0,1)$ having

   a) no fixed point b) unique fixed point
5. State the implicit function theorem.
6. Is $\gamma(t) = (t^2, t^4)$ a parametrization of the parabola $y = x^2$? Justify your answer.
7. Show that the curve $\gamma(t) = \left(\frac{4}{5}cost, 1 - sint, -\frac{3}{5}cost\right)$ has unit speed.
8. Define a regular curve. Is the curve $\gamma(t) = (e^{kt}\cos t, e^{kt}\sin t)$regular?
9. Compute the curvature of the curve $\gamma(t) = (t, cosht)$
10. Define a smooth surface in R^3. Prove that the unit sphere in R^3 given by $S^2 = \{(x, y, z)/x^2 + y^2 + z^2 = 1\}$ is a smooth surface.
11. Calculate the first fundamental form of the surface whose surface patch is given by $\sigma(u, v) = (\sinh u \sinh v, \ \sinh u \cosh v, \ \sinh u)$
12. Prove that for a plane, the unit normal is constant.
13. Define $\mathcal{G}auss\ map$.
14. If the Principal curvatures of a surface are 2 and 3 respectively, then find its Mean curvature and Gaussian curvature.

($14 \times 1 = 14$ Weightage)

PART B

Answer any s***even*** questions. Each question carries 2 weightage.

15. Prove that a linear operator A on a finite dimensional space is one to one if and only if the range of A is all of X.

16. Prove that $L(R^n, R^m)$ is a metric space.
17. Let A be a function from an open set $E \subset R^n \ to \ R^m$. Prove that the derivative of A if it exists, is unique.
18. Let X be a complete metric space and Φ be a contraction from $X \ into \ X$. Then prove that Φ has a unique fixed point in X
19. Let γ be a unit-speed curve in R^3 with constant curvature and zero torsion. Then prove that, γ is a parametrization of (part of) a circle.
20. Prove that if γ is a regular closed curve then a unit speed reparametrization of γ is also closed.
21. If $f: S \to \tilde{S}$ is a smooth map between surfaces and p ∈ S,then prove that the derivative Map $D_p f: T_P S \to T_P \tilde{S}$ is a linear map.
22. Compute the second fundamental form of the surface $\sigma(u, v) = (u, v, u^2 + v^2)$
23. Prove that the Weingarten map is self adjoint.
24. Let S be a connected surface of which every point is an umbilic. Then, prove that S is an open subset of a plane or a sphere.

($7 \times 2 = 14$ Weightage)

PART C

Answer any ***two*** questions. Each question carries 4 weightage.

25. a) Suppose f maps an open set $E \subset R^n$ into R^m, and f is differentiable at a point $x \in E$. Then prove that the partial derivatives $D_j f_i(x)$ $(1 \le j \le n, 1 \le i \le m)$ exist at all points of E

    b) If $f(0,0) = 0$ and $f(x, y) = \frac{xy}{x^2 + y^2}$ if $(x, y) \neq (0, 0)$, then prove that the function f is not differentiable in R^2 even though all the partial derivatives of f exist at all point of R^2
26. State and prove the Inverse function theorem.
27. Prove that a parametrized curve has a unit speed reparametrization if and only if it is regular.
28. Let $\sigma: U \to R^3$ be a surface patch. Let $(u_0, v_0) \in U$, and let $\delta > 0$ be such that the closed disc $R_\delta = \{(u, v) \in R^2 / (u - u_0)^2 + (v - v_0)^2 \le \delta^2\}$ with centre (u_0, v_0) and radius δ is contained in U. Then prove that $\lim_{\delta \to 0} \frac{\mathcal{A}_N(R_\delta)}{\mathcal{A}_\sigma(R_\delta)} = |K|$, where K is the Gaussian curvature of σ at (0, 0)

($2 \times 4 = 8$ Weightage)

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