18P370 (Pages: 2) Name……………………..

Reg. No…………………..

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(Regular/Supplementary/Improvement)

(CUCSS-PG)

CC15P ST3 C12 - TESTING OF STATISTICAL HYPOTHESES

(Statistics)

(2015 Admission onwards)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. What is $p -$ value? How is it used in statistical test procedure?
2. Define test function, type I error and type II error.
3. A sample of size 1 is taken from a Poisson distribution with parameter λ. Consider the following test for testing $H_0 : \lambda = 2$ against $H_1 : \lambda = 2$ where X denotes the sample.

$$\varphi(x) = \begin{cases} 1 \; if \; x \geq 3 \\ 0 \; if \; x < 3 \end{cases}$$

   Find the probability of type I error and power of the test.
4. Define locally Uniformly Most Powerful test.
5. What are Bayesian tests?
6. Define α- similar test.
7. Explain briefly chi-square test for homogeneity.
8. Define Kendall’s tau. State its properties.
9. Describe Kolmogorov-Smirnov two sample test.
10. Define test with Neyman structure.
11. Explain sequential estimation procedure.
12. Define:

    (i) OC function (ii) ASN function

(12 x 1 = 12 Weightage)

Part B

Answer any ***eight*** questions. Each question carries 2 weightage.

13. State and prove Neyman Pearson lemma.
14. Find Neyman Pearson size α test if $H_0\,;\,\theta = 1$ against $H_1\,;\,\theta = \theta_1 (> 1)$ based on a sample of size one from

$$f(x, \theta) = \begin{cases} \theta x^{\theta - 1}, 0 < x < 1 \\ 0 \;\; elsewhere \end{cases}$$

15. Show that the family of uniform densities on $[0,\ \theta]$ has MLR property.
16. State and prove the necessary and sufficient condition for all similar tests to have Neyman's structure test.
17. Define likelihood ratio test. Show the test is consistent.
18. Explain Mann-Whitney U – test for two sample problem.
19. Examine by sign test whether the following observations are coming from a population with median 25. Observations are 30.2, 25.3,27.9, 28.9, 23.3, 27.1, 22.4, 28.3, 24.0, 26.6, 28.3, 23,9, 27.1, 29.4, 28.1, 23.7
20. What is maximal invariant statistic? If $T(x)$ is maximal invariant with respect to $\mathcal{G}$ then prove that the test φ is invariant if and only if φ is a function of T
21. Define SPRT. How do you determine stopping bounds of SPRT
22. Consider the problem of testing $H_0\ ;\ \theta = \theta_0$ against $H_1\ ;\ \theta = \theta_1$ using random observations sequentially made on $X \sim B(1,\ \theta)$. Derive the SPRT for this testing problem.
23. Show that the SPRT terminates with probability one.
24. State and prove Wald' equation in sequential statistical inference.

(8 x 2 = 16 Weightage)

Part C

Answer any ***two*** questions. Each question carries 4 weightage.

25. a) Consider a population following $N(\mu,\ 5^2)$. Let $H_0 : \mu = 68$ against $H_1 : \mu = 69.$ and the critical region be $\overline{X} \geq k.$ where $\bar{X}$ is the sample mean. Find k and the sample size if the significance level $\alpha = 0.05$ and power of test = 0.95

    b) Show that the most powerful test of the Neyman-Pearson lemma for simple hypothesis against simple alternative is strictly unbiased, if $0 < \alpha < 1$
26. Show that the likelihood ratio test criterion for testing $H_0: \sigma_1^2 = \sigma_2^2$ against $H_0: \sigma_1^2 \neq \sigma_2^2$ where σ_1^2 and σ_2^2 are the variances of two population leads to F statistic.
27. Describe Wilcoxon Signed rank test. Find the probability distribution of the test statistic under the null hypothesis.
28. Explain Wald-Wolfomitiz run test. Describe its merits over median test.

(2 x 4 = 8 Weightage)

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